\[
\sin(2\theta) = 2 \sin \theta \cos \theta \\
\cos(2\theta) = \cos^2 \theta - \sin^2 \theta \\
e^x = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for } |x| < \infty \\
\ln(1 + x) = \sum_{k=1}^{\infty} \frac{(-1)^{k+1}x^k}{k}, \text{ for } -1 < x \leq 1 \\
\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}, \text{ for } |x| < \infty \\
\tan^{-1} x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{2k+1}, \text{ for } |x| \leq 1
\]

**Remainder Theorem:** Let \( R_n = |S - S_n| \) be the remainder in approximating the value of a convergent alternating series \( \sum_{k=1}^{\infty} (-1)^k a_k \) by the sum of its first \( n \) terms. Then \( R_n \leq a_{n+1} \).

1. **10 pts. each** If a series converges, use the Alternating Series Test to show it; otherwise, use some other test to show divergence.
   
   (a) \( \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k^2 + 4}} \)
   
   (b) \( \sum_{k=1}^{\infty} (-1)^{k+1} \frac{2k^2 + 3}{5k^2 + 1} \)

2. **10 pts.** Estimate the value of the convergent series \( \sum_{k=1}^{\infty} \frac{(-1)^k}{(2k + 1)^3} \) with an absolute error less than 10^{-3}.

3. **15 pts. each** Determine the interval of convergence and radius of convergence of the power series, making sure to test endpoints.
   
   (a) \( \sum_{k=0}^{\infty} \left( \frac{x - 1}{5} \right)^k \)
   
   (b) \( \sum_{k=1}^{\infty} \frac{(2x + 3)^k}{6k} \)

4. **10 pts.** Use the geometric series
   
   \[ f(x) = \frac{1}{1 - x} = \sum_{k=0}^{\infty} x^k, \quad |x| < 1 \]
   
   to find the power series representation (centered at 0) of the function \( g(x) = \frac{2}{1 - 4x} \). Give the interval of convergence of the new series.

5. **10 pts.** Find the function represented by the series \( \sum_{k=0}^{\infty} (\sqrt{x} - 7)^k \), and give the interval of convergence of the series.

6. Let \( f(x) = \cos(5x) \).
   
   (a) **10 pts.** Find the first four nonzero terms of the Maclaurin series for \( f \).
   
   (b) **5 pts.** Write the power series using summation notation.
   
   (c) **10 pts.** Determine the interval of convergence for the series.

7. **10 pts.** Evaluate \( \lim_{x \to 0} \frac{3 \tan^{-1} x - 3x + x^3}{x^5} \) using Taylor series.\(^1\)

8. **10 pts.** Use a Taylor series to approximate \( \int_0^{0.15} \frac{\sin x}{x} \, dx \), retaining as many terms as needed to ensure the error is less than 10^{-4}.

9. **10 pts.** Consider the parametric equations
   
   \[ x = (t + 1)^2, \quad y = t + 2; \quad -10 \leq t \leq 10. \]

   Eliminate the parameter to obtain an equation in \( x \) and \( y \).

10. **10 pts.** Give two alternative representations of the point \( (8, \frac{2\pi}{3}) \) in polar coordinates.

11. **15 pts.** Convert the equation \( r \cos \theta = \sin(2\theta) \) to Cartesian coordinates, and describe the resulting curve.

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\(^1\)Do not use L'Hôpital's Rule.