1. We have
   \[ f(2) = 2(2)^3 + 2 - 12 = 18 - 12 = 6, \]
   and from \( f'(x) = 6x^2 + 1 \) we find that \( f'(2) = 25 \neq 0 \). Now, clearly \( f \) is differentiable on \((-\infty, \infty)\), and since \( f' > 0 \) on \((-\infty, \infty)\) we conclude that \( f \) is everywhere increasing and therefore one-to-one. By the appropriate theorem we then obtain
   \[ (f^{-1})'(6) = \frac{1}{f'(2)} = \frac{1}{25}. \]

2. Let \( f_1 \) be the restriction of \( f \) to the interval \([4, \infty)\). That is, \( f_1(x) = f(x) \) for \( x \geq 4 \). Then \( f_1 \) is a one-to-one function and thus has an inverse \( f_1^{-1} \). To find \( f_1^{-1} \) set \( y = f_1(x) \), so that \( y = (x - 4)^2 \) for \( x \geq 4 \). Then
   \[ \sqrt{y} = |x - 4| = x - 4, \]
   whence \( x = 4 + \sqrt{y} \). Since \( y = f_1(x) \) if and only if \( x = f_1^{-1}(y) \), we obtain \( f_1^{-1}(y) = 4 + \sqrt{y} \).

   Next, let \( f_2 \) be the restriction of \( f \) to the interval \((-\infty, 4)\). That is, \( f_2(x) = f(x) \) for \( x \leq 4 \). Then \( f_2 \) is a one-to-one function and has an inverse \( f_2^{-1} \). To find \( f_2^{-1} \) set \( y = f_2(x) \), so that \( y = (x - 4)^2 \) for \( x \leq 4 \). Then
   \[ \sqrt{y} = |x - 4| = -(x - 4) = 4 - x, \]
   whence \( x = 4 - \sqrt{y} \). Since \( y = f_2(x) \) if and only if \( x = f_2^{-1}(y) \), we obtain \( f_2^{-1}(y) = 4 - \sqrt{y} \).

   We have now found that there are two (local) inverses associated with \( f \): the function \( f_1^{-1} \) given by
   \[ f_1^{-1}(y) = 4 + \sqrt{y} \]
   with \( \text{Dom}(f_1^{-1}) = \text{Ran}(f_1) = [0, \infty) \), and \( f_2^{-1} \) given by
   \[ f_2^{-1}(y) = 4 - \sqrt{y} \]
   with \( \text{Dom}(f_2^{-1}) = \text{Ran}(f_2) = [0, \infty) \).

3a. \( f'(x) = \frac{2e^{2x}}{e^{2x} + 3} \)

3b. \( \text{Dom}(g) = (0, \infty) \), and for all \( x > 0 \) we have
   \[ g(x) = x^{\ln(x^3)} = \exp\left(\ln(x^{\ln(x^3)})\right) = \exp(\ln(x^3) \ln(x)) = \exp(3\ln^2(x)), \]
   and thus
   \[ g'(x) = \exp(3\ln^2(x)) \cdot (3\ln^2(x))' = x^{\ln(x^3)} \cdot 6\ln(x) \cdot \frac{6x\ln(x^3) \ln(x)}{x} = \frac{6x\ln(x^3) \ln(x)}{x}. \]

3c. For \( x \) such that \( \tan(x) > 0 \) we have
   \[ h(x) = (\tan x)^{\cos x} = \exp(\ln((\tan x)^{\cos x})) = \exp(\cos x \cdot \ln(\tan x)). \]
and thus

\[ h'(x) = \exp(\cos x \cdot \ln(\tan x)) \cdot (\cos x \cdot \ln(\tan x))' \]
\[ = \exp(\cos x \cdot \ln(\tan x)) \cdot \left( \cos x \cdot \frac{\sec^2 x}{\tan x} - \sin x \cdot \ln(\tan x) \right) \]
\[ = (\tan x)^{\cos x} (\csc x - \ln(\tan x)^{\sin x}) \]

3d. \[ k'(x) = \frac{7}{(4 - x^5) \ln(3)} \cdot (4 - x^5)' = -\frac{35x^4}{(4 - x^5) \ln(3)} \]

3e. \[ \ell'(x) = \frac{1}{\sqrt{1 - (e^{-2x})^2}} \cdot (e^{-2x})' = -\frac{2e^{-2x}}{\sqrt{1 - e^{-4x}}} \]

3f. \[ p'(x) = -\frac{1}{1 + (\sqrt{x})^2} \cdot (\sqrt{x})' = -\frac{1}{1 + x} \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}(1 + x)} \]

4a. \[ \int (3e^{-8x} - 8e^{11x})\,dx = -\frac{3}{8}e^{-8x} - \frac{8}{11}e^{11x} + c \]

4b. \[ \int \frac{4}{3 - 10x}\,dx = -\frac{2}{5} \ln |3 - 10x| + c \]

4c. Let \( u = x^4 \), so by the Substitution Rule we replace \( x^3 \,dx \) with \( \frac{1}{4} \,du \) to get
\[ \int x^3 g(x^4)\,dx = \int \frac{1}{4} g(u)\,du = \frac{1}{4} \cdot \frac{9^u}{\ln(9)} + c = \frac{9^{x^4}}{4 \ln(9)} + c. \]

5a. Let \( u = \ln(x) \), so when \( x = 1 \) we have \( u = \ln(1) = 0 \), and when \( x = 3e \) we have \( u = \ln(3e) \). Now, by the Substitution Rule we replace \( \frac{1}{x} \,dx \) with \( du \) to get
\[ \int_0^{\ln(3e)} e^u \frac{e^u}{2} \,du = \left[ \frac{1}{2} e^u \right]_0^{\ln(3e)} = \frac{1}{2} (e^{\ln(3e)} - e^0) = \frac{3e - 1}{2}. \]

5b. We have
\[ 5 \int_2^{2\sqrt{3}} \frac{1}{z^2 + 2^2} \,dz = 5 \left[ \frac{1}{2} \tan^{-1} \left( \frac{z}{2} \right) \right]_2^{2\sqrt{3}} = \frac{5}{2} \left[ \tan^{-1} (\sqrt{3}) - \tan^{-1} (1) \right] = \frac{5}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} \right) = \frac{5\pi}{24}. \]
6. For all \( x > 0 \) we have
\[
\left( \frac{2}{3x} \right)^{8/x} = \exp \left[ \ln \left( \frac{2}{3x} \right)^{8/x} \right] = \exp \left[ \frac{8}{x} \ln \left( \frac{2}{3x} \right) \right] = \exp \left( \frac{8 \ln(2/3x)}{x} \right) .
\]
The functions \( f(x) = 8 \ln(2/3x) \) and \( g(x) = x \) are differentiable on \((0, \infty)\), and \( g'(x) = 1 \neq 0 \) for all \( x \in (0, \infty) \). Since \( g(x) \to \infty \) as \( x \to \infty \), and 
\[
\lim_{x \to \infty} \frac{f'(x)}{g'(x)} = \lim_{x \to \infty} \frac{-8/x}{1} = 0,
\]
by L’Hôpital’s Rule we obtain 
\[
\lim_{x \to \infty} \frac{f(x)}{g(x)} = \lim_{x \to \infty} \frac{8 \ln(2/3x)}{x} = 0
\]
as well. Now, since \( \exp(x) \) is a continuous function,
\[
\lim_{x \to \infty} \left( \frac{2}{3x} \right)^{8/x} = \lim_{x \to \infty} \exp \left( \frac{8 \ln(2/3x)}{x} \right) = \exp \left( \lim_{x \to \infty} \frac{8 \ln(2/3x)}{x} \right) = \exp(0) = 1.
\]

7. We find that \( x^{20}, 1.001^x \to \infty \) as \( x \to \infty \). To determine which grows faster, \( x^{20} \) or \( 1.001^x \), we must evaluate
\[
\lim_{x \to \infty} \frac{1.001^x}{x^{20}} .
\]
If the limit equals 0 then \( x^{20} \) grows faster, and if the limit equals \( \infty \), then \( 1.001^x \) grows faster. We have 
\[
\lim_{x \to \infty} \ln \left( \frac{1.001^x}{x^{20}} \right) = \lim_{x \to \infty} \left( \ln 1.001^x - \ln x^{20} \right) = \lim_{x \to \infty} \left( x \ln 1.001 - 20 \ln x \right)
\]
\[
= \lim_{x \to \infty} x \left( \ln 1.001 - \frac{20 \ln x}{x} \right),
\]
where we easily find by L’Hôpital’s Rule that
\[
\lim_{x \to \infty} \frac{20 \ln x}{x} = \lim_{x \to \infty} \frac{20/x}{1} = 0,
\]
so that
\[
\lim_{x \to \infty} \ln \left( \frac{1.001^x}{x^{20}} \right) = \ln 1.001 > 0
\]
and therefore
\[
\lim_{x \to \infty} \ln \left( \frac{1.001^x}{x^{20}} \right) = \lim_{x \to \infty} x \left( \ln 1.001 - \frac{20 \ln x}{x} \right) = (\infty)(\ln 1.001) = \infty.
\]
Now,
\[
\lim_{x \to \infty} \frac{1.001^x}{x^{20}} = \lim_{x \to \infty} \exp \left[ \ln \left( \frac{1.001^x}{x^{20}} \right) \right] = \exp \left[ \lim_{x \to \infty} \ln \left( \frac{1.001^x}{x^{20}} \right) \right] = \exp(\infty) = \infty,
\]
so \( 1.001^x \) grows faster than \( x^{20} \) and we write \( 1.001^x \gg x^{20} \).