1a. Clearly \( a_{k+1} = \frac{1}{\sqrt{(k+1)^2 + 4}} \leq \frac{1}{\sqrt{k^2 + 4}} = a_k \) for all \( k \), and \( \lim_{k \to \infty} \frac{1}{\sqrt{k^2 + 4}} = 0 \), so \( \sum_{k=1}^{\infty} (-1)^k \frac{1}{\sqrt{k^2 + 4}} \) converges by the Alternating Series Test.

1b. Diverges by the Divergence Test since \( \lim_{k \to \infty} \left| (-1)^{k+1} \frac{2k^2 + 3}{5k^2 + 1} \right| = \lim_{k \to \infty} \frac{2k^2 + 3}{5k^2 + 1} = \frac{2}{5} \neq 0 \) implies that \( \lim_{k \to \infty} (-1)^{k+1} \frac{2k^2 + 3}{5k^2 + 1} \neq 0 \).

2. We have \( \frac{1}{(2k+1)^3} < 10^{-3} \) when \( (2k+1)^3 > 1000 \), which occurs when \( 2k+1 > 10 \). Solving, we arrive at \( k > 4.5 \), or in our case \( k = 5 \) since \( k \) must be an integer. By the Remainder Theorem, then, the error will be less than \( 10^{-3} \) if we estimate the series by \( \sum_{k=1}^{3} (-1)^k/(2k+1)^3 = -1/3^3 + 1/5^3 - 1/7^3 + 1/9^3 \approx -0.0306 \).

3a. Applying Ratio Test, \( \lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lim_{k \to \infty} \frac{(x-1)^{k+1}}{5^k(x-1)^k} = \lim_{k \to \infty} \frac{|x-1|}{5} = \frac{|x-1|}{5} \), so series converges if \( \frac{|x-1|}{5} < 1 \), implying \(-4 < x < 6 \). When \( x = 6 \), \( \lim_{k \to \infty} \frac{(x-1)^k}{5^k} = \lim_{k \to \infty} \frac{6-1}{5} = 1 \neq 0 \), so series diverges by Divergence Test. When \( x = -4 \), \( \lim_{k \to \infty} \frac{(x-1)^k}{5^k} = \lim_{k \to \infty} \frac{-4-1}{5} = \lim_{k \to \infty} (-1)^k \neq 0 \), so again the series diverges. Interval of convergence is \((-4, 6)\), radius of convergence is 5.

3b. Applying Ratio Test, \( \lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lim_{k \to \infty} \frac{(2x+3)^{k+1}}{6(k+1)^3} = \lim_{k \to \infty} \frac{2x+3}{k+1} = |2x+3| \), so series converges if \(-1 < 2x+3 < 1 \), implying \(-2 < x < -1 \). When \( x = -2 \) series becomes \( \sum_{k=1}^{\infty} \frac{(-1)^k}{6k} \), which converges by the Alternating Series Test. When \( x = -1 \) series becomes \( \sum_{k=1}^{\infty} \frac{1}{6k} \), which diverges since \( \sum_{k=1}^{\infty} \frac{1}{k} \) diverges. Interval of convergence is \([-2, -1)\), radius of convergence is \(\frac{1}{2}\).

4. \( g(x) = \sum_{k=0}^{\infty} 2 \cdot (4x)^k \), which converges when \(|4x| < 1 \), so the interval of convergence is \((-\frac{1}{4}, \frac{1}{4})\).

5. Use the geometric series given in the previous problem to get \( f(x) = \frac{1}{1 - (\sqrt{x} - 7)} = \frac{1}{8 - \sqrt{x}} \). Series converges when \(|\sqrt{x} - 7| < 1 \), which solves to give \( 6 < \sqrt{x} < 8 \) and then \( 36 < x < 64 \). So interval of convergence is \((36, 64)\).

6a. \( \frac{5^0}{0!} x^0 - \frac{5^2}{2!} x^2 + \frac{5^4}{4!} x^4 - \frac{5^6}{6!} x^6 + \cdots = 1 - \frac{25}{2} x^2 + \frac{625}{24} x^4 - \frac{3125}{144} x^6 + \cdots \)

6b. \( \sum_{k=0}^{\infty} \frac{(-1)^k 5^{2k}}{(2k)!} x^{2k} = \sum_{k=0}^{\infty} \frac{(-1)^k (5x)^{2k}}{(2k)!} \)

6c. Use the Ratio Test to find the interval of convergence \((-\infty, \infty)\).
7. Using the Maclaurin series for \( \tan^{-1} \), \[ \lim_{x \to 0} \frac{3 \tan^{-1} x - 3x + x^3}{x^5} = \lim_{x \to 0} \frac{3 \left( x - \frac{1}{3}x^3 + \frac{1}{5}x^5 - \frac{1}{7}x^7 + \cdots \right) - 3x + x^3}{x^5} = \lim_{x \to 0} \frac{3}{5}x^5 - \frac{3}{7}x^7 + \cdots = \lim_{x \to 0} \frac{3}{5} - \frac{3}{7}x^2 + \cdots = \frac{3}{5} \]

8. This is #10.4.35 in the book, which was actually done in class before the exam. Answer: 0.1498.

9. From \( y = t + 2 \) we get \( t = y - 2 \), and then \( x = (t + 1)^2 \) becomes \( x = (y - 1)^2 \). Note that \( t \in [-10, 10] \) implies that \( y \in [-8, 12] \), so only part of a parabola results.

10. \((-8, -\pi/3)\) and \((8, -4\pi/3)\).

11. The first thing to notice is that any point where \( \theta = \pi/2 \) will satisfy the equation, which corresponds to the vertical line \( x = 0 \). Assuming we’re not on this line, we have \( x \neq 0 \) and thus \( r \neq 0 \), which then implies \( \cos \theta = x/r \) and \( \sin \theta = y/r \), and so (recalling \( r^2 = x^2 + y^2 \) and \( \sin 2\theta = 2 \sin \theta \cos \theta \)), we find that \( r \cos \theta = \sin(2\theta) \) \[ \Rightarrow x = \frac{2xy}{r^2} \Rightarrow x = \frac{2xy}{x^2 + y^2} \Rightarrow 1 = \frac{2y}{x^2 + y^2} \Rightarrow x^2 + (y - 1)^2 = 1 \]. This is a circle centered at \((0, 1)\) with radius 1. So, the graph of \( r \cos \theta = \sin(2\theta) \) is as pictured.