1. A little trial-and-error readily gives us

\[ g(1) = 1^5 - 1^3 + 2(1) = 2, \]

and so the Inverse Function Theorem, along with \( g'(x) = 5x^4 - 3x^2 + 2 \), implies

\[ (g^{-1})'(2) = \frac{1}{g'(1)} = \frac{1}{4}. \]

But can we use the theorem? Note that \( g'(1) = 4 > 0 \). Since \( g' \) is continuous and \( g'(1) > 0 \), we in fact must have \( g'(x) > 0 \) for all \( x \) in some open interval \( I \) containing 1, meaning \( g \) is strictly increasing on \( I \), and hence \( g \) is one-to-one on \( I \). Therefore \( g : I \to g(I) \) has an inverse function \( g^{-1} : g(I) \to I \) which, along with the differentiability of \( g \) on \( I \), allows us to use the Inverse Function Theorem in the manner above.

2a. \( f'(x) = \frac{2e^{2x}}{e^{2x} + 3} \)

2b. \( \text{Dom}(g) = (0, \infty) \), and for all \( x > 0 \) we have

\[ g(x) = x^{\ln(x^5)} = \exp\left(\ln\left(x^{\ln(x^5)}\right)\right) = \exp(\ln(x^5) \ln(x)) = \exp(5 \ln^2(x)), \]

and thus

\[ g'(x) = \exp(5 \ln^2(x)) \cdot (5 \ln^2(x))' = x^{\ln(x^5)} \cdot \frac{10 \ln(x)}{x} = \frac{10x^{\ln(x^5)} \ln(x)}{x}. \]

2c. For \( x \) such that \( \sin x > 0 \) we have

\[ h(x) = (\sin x)^{\tan x} = \exp(\ln((\sin x)^{\tan x})) = \exp(\tan x \cdot \ln(\sin x)), \]

and thus

\[ h'(x) = \exp(\tan x \cdot \ln(\sin x)) \cdot (\tan x \cdot \ln(\sin x))' \]
\[ = \exp(\tan x \cdot \ln(\sin x)) \cdot \left(\tan x \cdot \frac{\cos x}{\sin x} + \sec^2 x \cdot \ln(\sin x)\right) \]
\[ = (\sin x)^{\tan x} \left(1 + \ln(\sin x)\sec^2 x\right) \]

2d. \( k'(x) = \frac{7}{(4 - x^5) \ln(3)} \cdot (4 - x^5)' = -\frac{35x^4}{(4 - x^5) \ln(3)} \)

2e. \( \ell'(x) = \frac{1}{e^{-2x} \sqrt{(e^{-2x})^2 - 1}} \cdot (e^{-2x})' = -\frac{-2e^{-2x}}{e^{-2x} \sqrt{e^{-4x} - 1}} = -\frac{2}{\sqrt{e^{-4x} - 1}} \)
\[ p'(x) = -\frac{1}{1 + (\sqrt{x})^2} \cdot (\sqrt{x})' = -\frac{1}{1 + x} \cdot \frac{1}{2\sqrt{x}} = -\frac{1}{2\sqrt{x}(1 + x)} \]

\[ \int (3e^{-8x} - 8e^{11x}) \, dx = -\frac{3}{8}e^{-8x} - \frac{8}{11}e^{11x} + c \]

\[ \int \frac{9}{4 - 9y} \, dy = -\ln |4 - 9y| + c \]

Let \( u = x^8 \), so by the Substitution Rule we replace \( x^7 \, dx \) with \( \frac{1}{8} \, du \) to get

\[ \int x^7 8^x \, dx = \frac{1}{8} \int 8^u \, du = \frac{1}{8} \cdot \frac{8^u}{\ln 8} + c = \frac{8^{x^8}}{8\ln 8} + c. \]

Let \( u = \ln(x) \), so when \( x = 1 \) we have \( u = \ln(1) = 0 \), and when \( x = 3e \) we have \( u = \ln(3e) \).

Now, by the Substitution Rule we replace \( \frac{1}{x} \, dx \) with \( du \) to get

\[ \int_{\ln(3e)}^{\infty} \frac{e^u}{2} \, du = \left[ \frac{1}{2}e^u \right]_{\ln(3e)}^{\infty} = \frac{1}{2}(e^{\ln(3e)} - e^0) = \frac{3e - 1}{2}. \]

We have

\[ 5 \int_2^{2\sqrt{3}} \frac{1}{z^2 + 2^2} \, dz = 5 \left[ \frac{1}{2} \tan^{-1}\left( \frac{z}{2} \right) \right]^{2\sqrt{3}}_2 = \frac{5}{2} \left[ \tan^{-1}(\sqrt{3}) - \tan^{-1}(1) \right] = \frac{5}{2} \left( \frac{\pi}{3} - \frac{\pi}{4} \right) = \frac{5\pi}{24}. \]

For \( x \) near 0 but not equal to 0, for instance for \( x \in I = (-\frac{1}{4}, 0) \cup (0, \frac{1}{4}) \), we have

\[ \lim_{x \to 0} (x + \cos x)^{1/3x} = \exp \left[ \ln(x + \cos x)^{1/3x} \right] = \exp \left[ \frac{\ln(x + \cos x)}{3x} \right]. \]

The functions \( f(x) = \ln(x + \cos x) \) and \( g(x) = 3x \) are differentiable on \( I \), with \( f(x)/g(x) \to 0/0 \) as \( x \to 0 \). Since

\[ \lim_{x \to 0} \frac{f'(x)}{g'(x)} = \lim_{x \to 0} \frac{1 - \sin x}{3(x + \cos x)} = \frac{1}{3}, \]

by L'Hôpital's Rule it follows that

\[ \lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{\ln(x + \cos x)}{3x} = \frac{1}{3} \]

as well. Now, since \( \exp(x) \) is a continuous function,

\[ \lim_{x \to 0} (x + \cos x)^{1/3x} = \lim_{x \to 0} \exp \left[ \frac{\ln(x + \cos x)}{3x} \right] = \exp \left[ \lim_{x \to 0} \frac{\ln(x + \cos x)}{3x} \right] = \exp \left( \frac{1}{3} \right) = e^{1/3}. \]
6 Using the Chain Rule yields
\[ f'(x) = \frac{1}{2} (\tanh 5x)^{-1/2} \cdot \text{sech}^2 5x \cdot 5 = \frac{5 \text{sech}^2 5x}{2 \sqrt{\tanh 5x}}. \]

7 Make the substitution \( u = \sqrt{x} \), and then \( w = \cosh u \):
\[
\int_1^4 \frac{\tanh \sqrt{x}}{\sqrt{x}} \, dx = \int_1^2 2 \tanh u \, du = 2 \int_1^2 \frac{\sinh u}{\cosh u} \, du = 2 \int_{\cosh 1}^{\cosh 2} \frac{1}{w} \, dw = 2 \left[ \ln |w| \right]_{\cosh 1}^{\cosh 2}
\]
\[
= 2 \ln \left( \frac{\cosh 2}{\cosh 1} \right) = 2 \ln \left( \frac{e^2 + e^{-1}}{2} \cdot \frac{2}{e + e^{-1}} \right) = 2 \ln \left( \frac{e^4 + 1}{e^4 + e^2} \right)
\]
\[
= 2 \ln \left( \frac{e^4 + 1}{e^2 + 1} \right) - 4.
\]