1a Applying the Ratio Test,
\[
\lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \to \infty} \left| \frac{(x + 1)^{k+1}}{8^{k+1}} \cdot \frac{8^k}{(x + 1)^k} \right| = \lim_{k \to \infty} \left| \frac{x + 1}{8} \right| = \frac{|x + 1|}{8},
\]
so the series converges if \(|x + 1|/8 < 1\), implying \(-8 < x + 1 < 8\) and thus \(-9 < x < 7\). It remains to test the endpoints.

When \(x = 7\) the series becomes,
\[
\lim_{k \to \infty} \left( \frac{x + 1}{8} \right)^k = \lim_{k \to \infty} \left( \frac{7 + 1}{8} \right)^k = \lim_{k \to \infty} (1) = 1 \neq 0,
\]
so the series diverges by the Divergence Test.

When \(x = -9\) the series becomes,
\[
\lim_{k \to \infty} \left( \frac{x + 1}{8} \right)^k = \lim_{k \to \infty} \left( \frac{-9 + 1}{8} \right)^k = \lim_{k \to \infty} (-1)^k \neq 0,
\]
so again the series diverges. Therefore the interval of convergence is \((-9, 7)\), and the radius of convergence is \(|−9 − 7|/2 = 8\).

1b Applying the Ratio Test,
\[
\lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \to \infty} \left| \frac{(2x + 3)^{k+1}}{6(k + 1)} \cdot \frac{6k}{(2x + 3)^k} \right| = \lim_{k \to \infty} \left| \frac{k|2x + 3|}{k + 1} \right| = |2x + 3|,
\]
so the series converges if \(-1 < 2x + 3 < 1\), implying \(-2 < x < -1\).

When \(x = -2\) the series becomes
\[
\sum_{k=1}^{\infty} \frac{(-1)^k}{6k},
\]
which converges by the Alternating Series Test. When \(x = -1\) the series becomes
\[
\sum_{k=1}^{\infty} \frac{1}{6k},
\]
which diverges since
\[
\sum_{k=1}^{\infty} \frac{1}{k}
\]
diverges. Interval of convergence is \([-2, -1)\), radius of convergence is \(\frac{1}{2}\).

1c Clearly the series converges when \(x = -2\). Assuming \(x \neq -2\), we can employ the Ratio Test with
\[
a_k = (-1)^k \frac{(x + 2)^k}{k \cdot 2^k}
\]
to obtain
\[
\lim_{k \to \infty} \left| \frac{a_{k+1}}{a_k} \right| = \lim_{k \to \infty} \left| \frac{(-1)^{k+1}(x + 2)^{k+1}}{(k + 1) \cdot 2^{k+1}} \cdot \frac{k \cdot 2^k}{(-1)^k(x + 2)^k} \right|
\]
Thus the series converges if \( \frac{1}{2} |x + 2| < 1 \), which implies \( |x + 2| < 2 \) and thus \(-4 < x < 0 \). The Ratio Test is inconclusive when \( x = -4 \) or \( x = 0 \), so we analyze these endpoint separately.

When \( x = -4 \) the series becomes
\[
\sum_{k=0}^{\infty} \frac{(-1)^k(-2)^k}{k \cdot 2^k} = \sum_{k=0}^{\infty} \frac{2^k}{k \cdot 2^k} = \sum_{k=0}^{\infty} \frac{1}{k},
\]
which is the harmonic series and therefore diverges.

When \( x = 0 \) the series becomes
\[
\sum_{k=0}^{\infty} \frac{(-1)^k 2^k}{k \cdot 2^k} = \sum_{k=0}^{\infty} (-1)^k \frac{1}{k},
\]
which is an alternating series \( \sum (-1)^k b_k \) with \( b_k = 1/k \). Since \( \lim_{k \to \infty} b_k = 0 \) and
\[
b_{k+1} = \frac{1}{k+1} < \frac{1}{k} = b_k
\]
for all \( k \), by the Alternating Series Test this series converges.

Therefore the series converges on the interval \((-4, 0]\), and the radius of convergence is \( R = \frac{1}{2} |0 - (-4)| = 2 \).

2 We manipulate to obtain
\[
h(x) = 2 \cdot \frac{1}{1 - (-3x)} = 2 \sum_{k=0}^{\infty} (-3x)^k = \sum_{k=0}^{\infty} 2(-3x)^k,
\]
which converges if and only if \( |-3x| < 1 \), so the interval of convergence is \((-\frac{1}{3}, \frac{1}{3})\).

3 Formally, the function represented by the series is given by
\[
f(x) = \frac{1}{1 - (\sqrt{x} + 4)} = -\frac{1}{3 + \sqrt{x}}.
\]
The series converges if and only if \( |\sqrt{x} + 4| < 1 \), or equivalently \(-5 < \sqrt{x} < -3 \). But there exists no \( x \in \mathbb{R} \) which satisfies this inequality, and so there is no interval of convergence!

4a We have
\[
3x - \frac{3^3}{3!} x^3 + \frac{3^5}{5!} x^5 - \frac{3^7}{7!} x^7 + \cdots = 3x - \frac{9}{2} x^3 + \frac{625}{24} x^5 - \frac{243}{560} x^7 + \cdots
\]

4b We have
\[
\sum_{k=0}^{\infty} \frac{(-1)^k (3x)^{2k+1}}{(2k + 1)!}
\]
4c Use the Ratio Test to find that the interval of convergence is \((-\infty, \infty)\).

5 We have
\[
\sin(x) = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}
\]
for all \(x \in (-\infty, \infty)\), and so
\[
\sin(x^2) = \sum_{k=0}^{\infty} \frac{(-1)^k (x^2)^{2k+1}}{(2k+1)!} = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!}
\]
(1)
for all \(-\infty < x < \infty\). In particular the series at right in (1) converges on \((-\infty, \infty)\), and so
\[
\int \left[ \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!} \right] dx = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+3}}{(4k+3)(2k+1)!} + c
\]
for all \(x \in (-\infty, \infty)\) and arbitrary constant \(c\). Thus, by the Fundamental Theorem of Calculus,
\[
\int_0^{0.2} \sin(x^2) \, dx = \int_0^{0.2} \left[ \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!} \right] dx = \left[ \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+3}}{(4k+3)(2k+1)!} \right]_0^{0.2}
\]
\[
= \sum_{k=0}^{\infty} \frac{(-1)^k (0.2)^{4k+3}}{(4k+3)(2k+1)!} - \sum_{k=0}^{\infty} \frac{(-1)^k (0)^{4k+3}}{(4k+3)(2k+1)!}
\]
\[
= \sum_{k=0}^{\infty} \frac{(-1)^k 0.2^{4k+3}}{(4k+3)(2k+1)!}
\]
We have arrived at an alternating series \(\sum (-1)^k b_k\) with
\[
b_k = \frac{0.2^{4k+3}}{(4k+3)(2k+1)!}
\]
for \(k \geq 0\). Evaluating the first few \(b_k\) values,
\[
b_0 = \frac{0.2^3}{(3 \cdot 3!)} \approx 2.6667 \times 10^{-3}
\]
\[
b_1 = \frac{0.2^7}{(7 \cdot 3!)} \approx 3.0476 \times 10^{-7}
\]
\[
b_2 = \frac{0.2^{11}}{(11 \cdot 5!)} \approx 1.5515 \times 10^{-11}
\]
By the Alternating Series Estimation Theorem the approximation
\[
\sum_{k=0}^{\infty} (-1)^k \frac{0.2^{4k+3}}{(4k+3)(2k+1)!} \approx b_0 - b_1 = \frac{0.2^3}{3} - \frac{0.2^7}{42}
\]
will have an absolute error that is less than \(b_2 \approx 1.5515 \times 10^{-11} < 10^{-10}\). Therefore the approximation
\[
\int_0^{0.2} \sin(x^2) \, dx \approx \frac{0.2^3}{3} - \frac{0.2^7}{42} \approx 0.002666
\]
has an absolute error less than \(10^{-10}\).
6. From $x = \sqrt{t} + 4$ comes $t = (x - 4)^3$. Putting this into $y = 5t - 3$ gives $y = 5(x - 4)^3 - 3$. That is, the function $f$ is given by

$$f(x) = 5(x - 4)^3 - 3.$$ 

From $t \in [0, 27]$ we find that $x \in [4, 7]$, so the domain of $f$ is $[4, 7]$.

7. $(2, 4\pi/3), (2, -2\pi/3), (-2, \pi/3)$, among other possibilities.

8. We have $r = f(\theta)$ with $f(\theta) = 8 \cos \theta$. The slope $m$ of the curve at $(4, 5\pi/6)$ is

$$m = \frac{f'(5\pi/6) \sin(5\pi/6) + f(5\pi/6) \cos(5\pi/6)}{f'(5\pi/6) \cos(5\pi/6) - f(5\pi/6) \sin(5\pi/6)}$$

$$= \frac{-8 \sin(5\pi/6) \sin(5\pi/6) + 8 \cos(5\pi/6) \cos(5\pi/6)}{-8 \sin(5\pi/6) \cos(5\pi/6) - 8 \cos(5\pi/6) \sin(5\pi/6)}$$

$$= \frac{\sin^2(5\pi/6) - \cos^2(5\pi/6)}{2 \cos(5\pi/6) \sin(5\pi/6)} = \frac{(1/2)^2 - (\sqrt{3}/2)^2}{2(1/2)(\sqrt{3}/2)} = \frac{1}{\sqrt{3}}$$

9. Here $r = f(\theta)$ with $f(\theta) = 3 + 5 \cos \theta$. The curve is generated for $\theta \in [0, 2\pi)$, so we find all $0 \leq \theta < 2\pi$ for which

$$f'(\theta) \sin \theta + f(\theta) \cos \theta = 0,$$

which gives

$$-5 \sin \theta \sin \theta + \cos \theta (3 + 5 \cos \theta) = 0.$$ 

Since $\sin^2 \theta = 1 - \cos^2 \theta$, we get

$$10 \cos^2 \theta + 3 \cos \theta - 5 = 0,$$

and so

$$\cos \theta = \frac{-3 \pm \sqrt{209}}{20} \approx -0.8728, 0.5728$$

by the quadratic formula. From $\cos \theta = -0.8728$ we obtain $\theta \approx 2.63, 3.65$. From $\cos \theta = 0.5728$ we obtain $\theta \approx 0.96, 5.32$. Solution set in $[0, 2\pi)$ is thus

$$\{0.96, 2.63, 3.65, 5.32\},$$

to the nearest hundredth.