1. The Root Test will do nicely here: \( \lim_{k \to \infty} \sqrt[k]{|a_k|} = \lim_{k \to \infty} \left| \frac{x - 1}{k+1} \right|^{1/k} = \lim_{k \to \infty} \frac{k}{k+1} \cdot |x - 1| = |x - 1| \), so the series will converge if \( |x - 1| < 1 \) (and diverge if \( |x - 1| > 1 \)), implying the radius of convergence is \( R = 1 \). That is, the series converges for all \( x \in (0, 2) \), and it remains to investigate the endpoints. When \( x = 2 \) the series becomes \( \sum_{k=0}^{\infty} \frac{k^k}{(k+1)^k} \), but since \( \lim_{k \to \infty} \frac{k^k}{(k+1)^k} = 1/e \neq 0 \), the series diverges by the Divergence Test. When \( x = 0 \) the series becomes \( \sum_{k=0}^{\infty} (-1)^k k^k \), but again \( \lim_{k \to \infty} \frac{(-1)^k k^k}{(k+1)^k} \neq 0 \) so the series diverges. Therefore the interval of convergence is \( (0, 2) \).

2. We use \( \sum_{k=0}^{\infty} \frac{1}{1 + x^2} = \sum_{k=0}^{\infty} (-1)^k (x^2)^k = \sum_{k=0}^{\infty} (-1)^k x^{2k} \). Now, \( \lim_{k \to \infty} \sqrt[k]{|a_k|} = \lim_{k \to \infty} \left| x^{2k} \right|^{1/k} = \lim_{k \to \infty} x^2 = x^2 \), so by the Root Test the series converges if \( x^2 < 1 \), which gives \( -1 < x < 1 \). If \( x = \pm 1 \) the series becomes \( \sum_{k=0}^{\infty} (-1)^k \), which diverges by the Divergence Test. Hence the interval of convergence is \( (-1, 1) \).

3a. From \( f(x) = e^{-3x} \), \( f'(x) = -3e^{-3x} \), \( f''(x) = 9e^{-3x} \), \( f'''(x) = -27e^{-3x} \) we get
\[
\frac{f(0) + f'(0)}{1!} x + \frac{f''(0)}{2!} x^2 + \frac{f'''(0)}{3!} x^3 + \cdots = 1 - 3x + \frac{9}{2} x^2 - \frac{9}{2} x^3 + \cdots
\]

3b. Power series for \( e^{-3x} \) is \( \sum_{k=0}^{\infty} \frac{(-1)^k 3^k}{k!} x^k \)

3c. Use Ratio Test: \( \lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lim_{k \to \infty} \left| \frac{(-1)^{k+1} 3^{k+1} (k+1)!}{(k+2)!} \cdot \frac{k!}{(-1)^k 3^k x^k} \right| = \lim_{k \to \infty} \left| \frac{3x}{k+1} \right| = 0 \) for all \( x \in \mathbb{R} \), so the interval of convergence is \( (-\infty, \infty) \).

4. Use \( \sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \) to get \( \sin x^2 = \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!} \), which converges for all \( x \in \mathbb{R} \). Hence integration can be done “termwise” in the natural way:
\[
\int_0^{0.2} \sin x^2 \, dx = \int_0^{0.2} \sum_{k=0}^{\infty} \frac{(-1)^k x^{4k+2}}{(2k+1)!} \, dx = \sum_{k=0}^{\infty} \left[ \frac{(-1)^k}{(2k+1)!} \right]_0^{0.2} \frac{x^{4k+2}}{4k+3} = \sum_{k=0}^{\infty} \frac{(-1)^k}{(2k+1)!} \frac{0.2^{4k+3}}{4k+3} = \frac{1}{375} - 3.048 \times 10^{-7} + \cdots.
\]
The series is a convergent alternating series, so the error in estimating its value by taking the first \( n \) terms will be less than the value of the \((n+1)\)st term. So here, to estimate the value of the series with an error less than \( 10^{-4} \), we only need the first term: \( 1/375 \).

5. From \( y = t + 2 \) we get \( t = y - 2 \), and then \( x = (t + 1)^2 \) becomes \( x = (y - 1)^2 \).

6. \((-3, -\pi/3)\) and \((3, -4\pi/3)\).
7. The first thing to notice is that any point where \( \theta = \pi/2 \) will satisfy the equation, which corresponds to the vertical line \( x = 0 \). Assuming we’re not on this line, we have \( x \neq 0 \) and thus \( r \neq 0 \), which then implies \( \cos \theta = x/r \) and \( \sin \theta = y/r \), and so (recalling \( r^2 = x^2 + y^2 \) and \( \sin 2\theta = 2 \sin \theta \cos \theta \)), we find that \( r \cos \theta = \sin(2\theta) \)

\[ \Rightarrow \quad x = \frac{2xy}{r^2} \Rightarrow x = \frac{2xy}{x^2 + y^2} \Rightarrow 1 = \frac{2y}{x^2 + y^2} \Rightarrow x^2 + (y - 1)^2 = 1. \]

This is a circle centered at \((0, 1)\) with radius 1. So, the graph of \( r \cos \theta = \sin(2\theta) \) is as pictured.