1. 243 and 729. Recurrence relation: \( a_{n+1} = 3a_n, \ a_1 = 1. \) Explicit formula: \( a_n = 3^{n-1} \) for \( n \geq 1. \)

2. We obtain \( \lim_{n \to \infty} (1 + 5/n)^n = \lim_{n \to \infty} \exp \left[ \ln(1 + 5/n)^n \right] = \exp \left[ \lim_{n \to \infty} \ln(1 + 5/n)^n \right] = \exp \left[ \lim_{n \to \infty} \frac{\ln(1 + 5/n)}{1/n} \right] \)

\[= \exp \left[ \lim_{n \to \infty} \frac{(1 + 5/n) - 1 - (-5/n^2)}{-1/n^2} \right] = \exp \left[ \lim_{n \to \infty} \frac{5}{1 + 5/n} \right] = \exp(5) = e^5, \]

using L'Hôpital's Rule en route.

3. \[ \sum_{k=2}^{\infty} \frac{1}{4^k} = \sum_{k=0}^{\infty} \frac{1}{4^{k+2}} = \sum_{k=0}^{\infty} \frac{1}{4^2} \cdot \left(\frac{1}{4}\right)^k = \frac{1/16}{1 - 1/4} = \frac{1}{12}. \]

4. \[ s_n = \sum_{k=1}^{n} \left( \frac{1}{k+2} - \frac{1}{k+3} \right) = \left( \frac{1}{3} - \frac{1}{4} \right) + \left( \frac{1}{4} - \frac{1}{5} \right) + \left( \frac{1}{5} - \frac{1}{6} \right) + \cdots + \left( \frac{1}{n+1} - \frac{1}{n+2} \right) + \left( \frac{1}{n+2} - \frac{1}{n+3} \right) \]

\[= \frac{1}{3} - \frac{1}{n+3}, \text{ so } \lim_{n \to \infty} s_n = \lim_{n \to \infty} \left( \frac{1}{3} - \frac{1}{n+3} \right) = \frac{1}{3}. \]

5. Here \( \lim_{k \to \infty} a_k = \lim_{k \to \infty} \frac{k^3}{k^3 + 10} = 1 \neq 0, \) and so the series diverges by the Divergence Test.

6. Let \( f(x) = xe^{-3x^2}, \) which clearly is continuous and nonnegative on \([1, \infty).\) Now, \( f'(x) = (1 - 6x^2)e^{-3x^2}, \) so we have \( f'(x) < 0 \) for \( x > 1/\sqrt{6} \) or \( x < -1/\sqrt{6}, \) which certainly shows that \( f \) is nonincreasing on \([1, \infty)\) as well. The hypotheses of the Integral Test are therefore satisfied for \( N = 1. \)

Making the substitution \( u = -3x^2, \) we obtain

\[ \int_1^\infty xe^{-3x^2} \, dx = \lim_{b \to \infty} \int_1^b xe^{-3x^2} \, dx = \lim_{b \to \infty} \int_{-3}^{-3b^2} \frac{1}{6} e^u \, du = \lim_{b \to \infty} \left[ \frac{1}{6} \left( e^{-3b^2} - e^{-3} \right) \right] = \frac{1}{6e^3}, \]

and thus the integral \( \int_1^\infty xe^{-3x^2} \, dx \) converges. Therefore by the Integral Test \( \sum_{k=1}^{\infty} ke^{-3k^2} \) converges as well.

7. Here \( a_k = \frac{k^6}{k!}, \) so \( r = \lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \lim_{k \to \infty} \frac{(k+1)^6}{k^6} \cdot \frac{k!}{(k+1)!} = \lim_{k \to \infty} \frac{(k+1)^5}{k^6} = 0 < 1 \) and by the Ratio Test the series converges.

8. \( r = \lim_{k \to \infty} \sqrt[k]{a_k} = \lim_{k \to \infty} \sqrt[k]{\left(\frac{k+1}{2k}\right)^k} = \lim_{k \to \infty} \frac{k+1}{2k} = \frac{1}{2} < 1, \) and so by the Root Test the series converges.

9. For all \( k \geq 1 \) we have \( |\sin(1/k)| \leq 1, \) and thus \( \left| \frac{\sin(1/k)}{k^2} \right| \leq \frac{1}{k^2}. \) Now, since the series \( \sum_{k=1}^{\infty} \frac{1}{k^2} \) converges (it's a \( p\)-series with \( p = 2), \) by the Comparison Test the series \( \sum_{k=1}^{\infty} \frac{\sin(1/k)}{k^2} \) converges.

10. It diverges by the Divergence Test, since the limit \( \lim_{k \to \infty} (-1)^k \frac{k^2 - 1}{k^2 + 3} \) does not exist (and therefore does not equal 0).