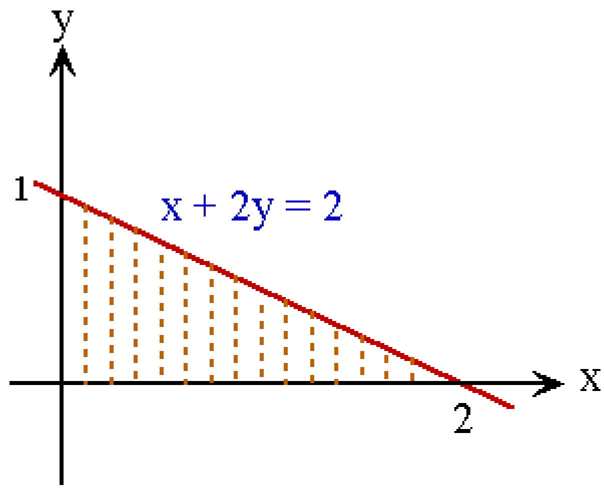
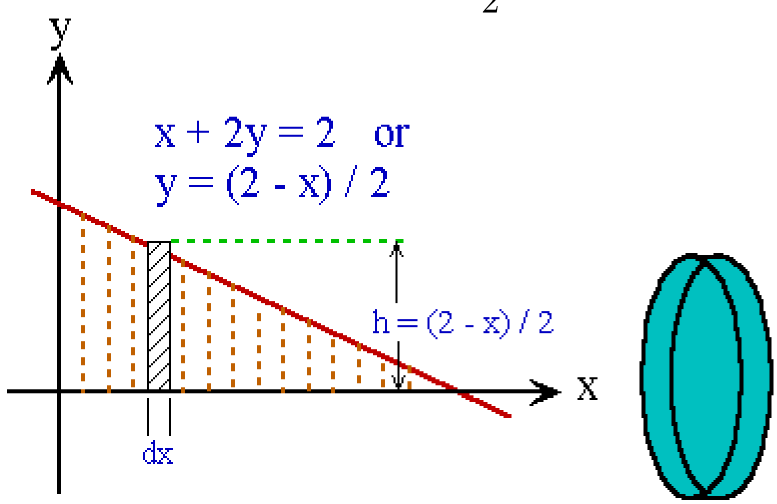
Thomas' Calculus
Tenth Edition

Section 5.1- Volumes by Slicing and Rotation About an Axis

13. Find the volume of the solid generated by revolving the shaded region about the x-axis.



Divide up the region into vertical rectangles and rotate the rectangles about the x-axis. Look at one rectangle. The height of this rectangle is $y = \dfrac{2-x}{2}$ and its width is dx .



When this rectangle is rotated about the x-axis you get a disk of radius $(2-x)/2$ and thickness dx.

Its volume is $dV = \pi\left(\frac{2-x}{2}\right)^2 dx$.

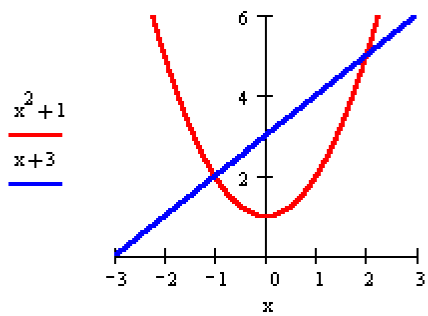
The total volume is $\pi\int_0^2 \left(\frac{2-x}{2}\right)^2 dx =$

$\frac{\pi}{4}\int_0^2 (4-4x+x^2)\,dx = \frac{\pi}{4}\left(4x-2x^2+\frac{x^3}{3}\right)\Big]_0^2 =$

$$\frac{\pi}{4}\left(8-8+\frac{8}{3}-0\right) = \mathbf{\frac{2\pi}{3}}.$$

35. Find the volume of the solid generated by revolving the region bounded by $y = x^2+1$ and $y = x+3$ about the x-axis.

The graphs of the functions are shown below:

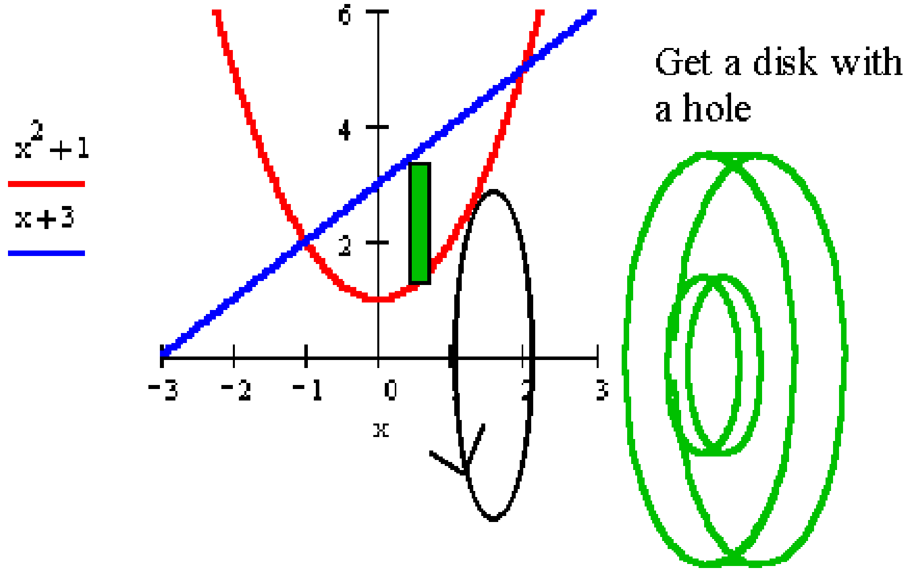


We must find where the 2 graphs intersect. To do this, see where

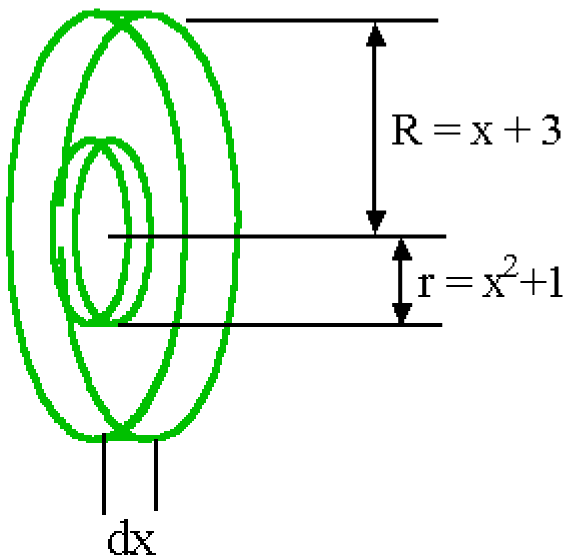
$x^2+1 = x+3 \Rightarrow x^2-x-2=0 \Rightarrow$

$(x-2)(x+1)=0 \Rightarrow x=2$ and $x=-1$.

Divide the area up into vertical rectangles and rotate one of the rectangles about the x-axis.



When the rectangle is revolved about the x-axis what results is a disk with a hole.



To get the volume of this disk with a hole, get the volume of the disk with the hole filled in and subtract the volume of the hole. This volume is $\pi R^2 dx - \pi r^2 dx \;=\; \pi(R^2 - r^2)dx$. Therefore

$$dV = \pi\Big((x+3)^2 - (x^2+1)^2\Big)dx =$$

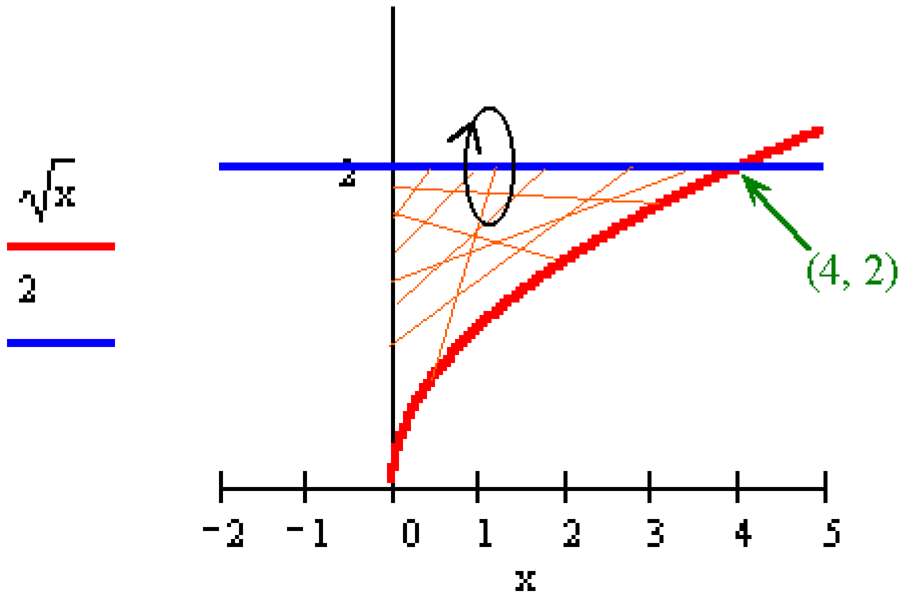
$$\pi\big(x^2 + 6x + 9 - (x^4 + 2x^2 + 1)\big) = \pi\big(-x^4 - x^2 + 6x + 8\big)$$

and the total volume is

$$\pi \int_{-1}^{2} \left(- x^4 - x^2 + 6x + 8 \right) dx = \mathbf{73.513}\,.$$

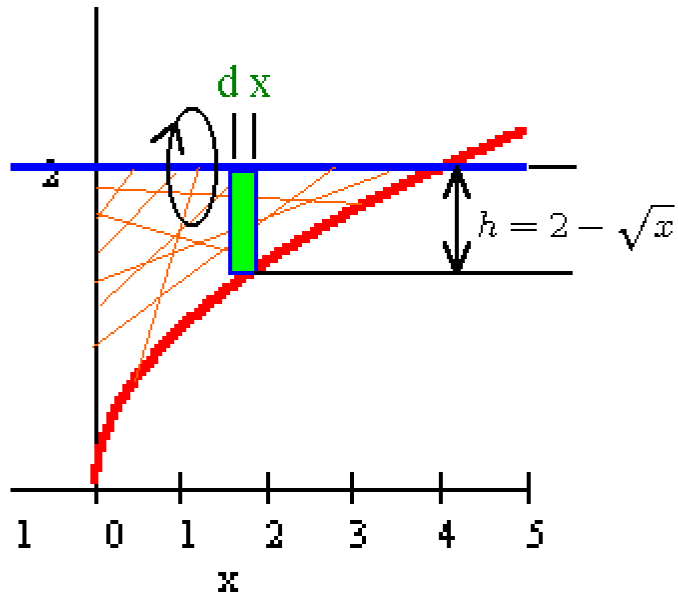
45. Find the volume of the solid generated by revolving the region bounded by $y = \sqrt{x}$ and the lines $y = 2$, $y = 0$, and $x = 0$ about
c) the line $y = 2$.

The area is shown below. Note that $x = 0$ is the y-axis.



The line $y = 2$ intersects the curve $y = \sqrt{x}$ when
$\sqrt{x} = 2 \;\Rightarrow\; \left(\sqrt{x}\right)^2 = 2^2 \;\Rightarrow\; x = 4$.

Divide the area up into vertical rectangles and see what happens when one of these rectangles is rotated about the line $y = 2$.



When a rectangle is rotated about the line $y = 2$ what results is a disk with radius $2 - \sqrt{x}$ and thickness dx. The volume of this disk is $dV = \pi\left(2 - \sqrt{x}\right)^2 dx$ and the total volume is

$$V = \pi\int_0^4 \left(2 - \sqrt{x}\right)^2 dx = \pi\int_0^4 \left(4 - 4\sqrt{x} + x\right)dx =$$

$$\pi\left(4x - 4\frac{x^{3/2}}{3/2} + \frac{x^2}{2}\right) = \pi\left(4x - \frac{8}{3}x^{3/2} + \frac{x^2}{2}\right)\Bigg]_0^4 =$$

$$\pi\left(16 - \frac{8}{3}\cdot 8 + 8\right) = \pi\left(24 - \frac{64}{3}\right) = \pi\left(\frac{72 - 64}{3}\right) = \mathbf{\frac{8\pi}{3}}.$$