

Finite Mathematics and Calculus with Applications
(7th Edition)
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Section 2.1- Solution of Linear Systems- Echelon Method

7. Use the echelon method to solve the system

$$\begin{aligned}2x - 3y &= -7 \\5x + 4y &= 17\end{aligned}$$

When we use this method, we rewrite the system in the form

$$\begin{aligned}x + by &= c \\y &= d\end{aligned}$$

Then the 2nd equation gives us the value of y and then we use this value in the 1st equation to get x .

Multiply the 1st equation by $1/2$.

$$\begin{aligned}x - 1.5y &= -3.5 \\5x + 4y &= 17\end{aligned}$$

Add -5 times the 1st equation to the 2nd equation.

$$\begin{aligned}-5x + 7.5y &= 17.5 \\5x + 4y &= 17\end{aligned}$$

$$11.5y = 34.5 \Rightarrow y = 34.5/11.5 = 3.$$

Therefore the system is equivalent to

$$\begin{aligned}x - 1.5y &= -3.5 \\y &= 3\end{aligned}$$

$$\text{and } x = 1.5y - 3.5 \Rightarrow x = 1.5(3) - 3.5 = 4.5 - 3.5 = 1.$$

Therefore the solution is $(1, 3)$.

21. Use the echelon method to solve the system

$$\begin{aligned}\frac{x}{2} + y &= \frac{3}{2} \\ \frac{x}{3} + y &= \frac{1}{3}\end{aligned}$$

Multiply the 1st equation by 2 and the 2nd by 3.

$$\begin{aligned}x + 2y &= 3 \\ x + 3y &= 1\end{aligned}$$

Add -1 times the 1st equation to the 2nd equation.

$$\begin{aligned}-x - 2y &= -3 \\ x + 3y &= 1\end{aligned}$$

$$\begin{array}{r} \text{-----} \\ y = -2 \end{array}$$

So the original system is equivalent to

$$\begin{aligned}x + 2y &= 3 \\ y &= -2\end{aligned}$$

and we know $y = -2$ and $x = 3 - 2y = 3 - 2(-2) = 7$.
The solution is $(7, -2)$.

23. Use the echelon method to solve the system

$$\begin{aligned}x + y + z &= 2 \\ 2x + y - z &= 5 \\ x - y + z &= -2\end{aligned}$$

Change the system to one of the form

$$\begin{aligned}x + by + cz &= d \\ y + ez &= f \\ z &= g\end{aligned}$$

Add -2 times the 1st equation to the 2nd equation and add -1 times the 1st equation to the 3rd equation. This results in

$$\begin{aligned}x + y + z &= 2 \\ -y - 3z &= 1\end{aligned}$$

$$-2y + 0z = -4$$

Multiply the 2nd equation by -1 .

$$\begin{aligned}x + y + z &= 2 \\y + 3z &= -1 \\-2y + 0z &= -4\end{aligned}$$

Add 2 times the 2nd equation to the 3rd equation.

$$\begin{aligned}x + y + z &= 2 \\y + 3z &= -1 \\6z &= -6\end{aligned}$$

Finally, multiply the 3rd equation by $-1/6$.

$$\begin{aligned}x + y + z &= 2 \\y + 3z &= -1 \\z &= -1\end{aligned}$$

This means $z = -1$,

$$y = -3z - 1 = 3 - 1 = 2,$$

$$x = 2 - y - z = 2 - 2 - (-1) = 1.$$

The solution is $(1, 2, -1)$.

29. Solve the system using the parameter z :

$$\begin{aligned}3x + y - z &= 0 \\2x - y + 3z &= -7\end{aligned}$$

Rewrite the system in the form

$$\begin{aligned}x + by + cz &= d \\y + ez &= f\end{aligned}$$

Add -1 times the 2nd equation to the 1st equation.

$$\begin{aligned}x + 2y - 4z &= 7 \\2x - y + 3z &= -7\end{aligned}$$

Add -2 times the 1st equation to the 2nd equation.

$$\begin{aligned}-2x - 4y + 8z &= -14 \\2x - y + 3z &= -7\end{aligned}$$

$$-5y + 11z = -21$$

The system is now:

$$\begin{aligned} x + 2y - 4z &= 7 \\ -5y + 11z &= -21 \end{aligned}$$

Finally, divide the 2nd equation by -5 .

$$\begin{aligned} x + 2y - 4z &= 7 \\ y - \frac{11}{5}z &= \frac{21}{5} \end{aligned}$$

So z is just z where z can be any number.

$$y = \frac{21}{5} + \frac{11}{5}z = \frac{21 + 11z}{5} \text{ and}$$

$$\begin{aligned} x = 7 - 2y + 4z &= \frac{35}{5} - \frac{42 + 22z}{5} + \frac{20}{5}z = \frac{35 - 42 - 22z + 20z}{5} \\ &= \frac{-7 - 2z}{5}. \end{aligned}$$

$$\text{The solution is } \left(\frac{-7 - 2z}{5}, \frac{21 + 11z}{5}, z \right).$$

37. A theater charges \$8 for main floor seats and \$5 for balcony seats. If all the seats are sold, the ticket income is \$4200. At one show, 25% of the main floor seats and 40% of the balcony seats were sold and ticket income was \$1200. How many seats are on the main floor and how many seats are on the balcony?

Let x be the number of seats on main floor and

let y be the number of seats on the balcony.

The information stated in the problem gives the 2 equations:

$$\begin{aligned} 8x + 5y &= 4200 \\ \frac{1}{4}(8x) + \frac{2}{5}(5y) &= 1200 \quad \leftarrow \frac{1}{4} = 25\% \text{ and } \frac{2}{5} = 40\% \end{aligned}$$

$$\begin{aligned} \text{The 2nd equation reduces to } 2x + 2y &= 1200 \text{ or} \\ x + y &= 600. \end{aligned}$$

So we have the 2 equations

$$\begin{aligned}x + y &= 600 \\8x + 5y &= 4200\end{aligned}$$

Add -8 times the 1st equation to the 2nd equation.

$$\begin{aligned}x + y &= 600 \\-3y &= -600 \text{ or } y = 200 . \\x &= 600 - y - 600 - 200 = 400 .\end{aligned}$$

Ans: There are 400 seats on the main floor and 200 on the balcony.