Practice for Test 4- Calculus II

Find the interval where the series converges. Where does it diverge? Graph the interval of convergence.

1) $\sum_{n=0}^{\infty} \frac{(x-9)^n}{4n+5}$

Answer: $8 \le x < 10$ Use ratio test. find where ratio is less than 1. Test the endpoints.

2) $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2 4^n}$

Answer: $-2 \le x \le 6$ see previous problem

3) $\sum_{n=1}^{\infty} \frac{(x-3)^n}{(2n)!}$

Answer: $-\infty < x < \infty$ The limit in the ratio test is 0. Since 0<1 for all reals the interval is all reals.

Find the Taylor polynomial of order 3 generated by f at a.

4) $f(x) = \ln x$, $a = 6$

Answer: $\ln 6 + \frac{x-6}{6} - \frac{(x-6)^2}{72} + \frac{(x-6)^3}{648}$

5) $f(x) = e^{-2x}$, $a = 0$

Answer: $1 - 2x + \frac{4x^2}{2} - \frac{8x^3}{6}$

Find the Maclaurin series. Begin with a series representing the basic series.

6) $\cos 9x$

Answer: $\sum_{n=0}^{\infty} \frac{(-1)^n 9^{2n} x^{2n}}{(2n)!}$ Find the series for cos x. Then substitute 9x for x.

7) $f(x) = x^5 \sin x$

Answer: $\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+6}}{(2n+1)!}$ Find the series for sinx. Then multiply by x^5.

Find the Cartesian coordinates corresponding to the given polar coordinates.

8) $(-4, -\pi/3)$

Answer: $(-2, 2\sqrt{3})$ Use $x = r\cos\theta$ and $y = r\sin\theta$

Find two sets of polar coordinates for the point with rectangular coordinates.

9) (2, 2)

Answer: $\left(2\sqrt{2}, \frac{\pi}{4}\right)$ and $\left(2\sqrt{2}, \frac{9\pi}{4}\right)$

10) Find the Maclaurin series for $f(x) = \cos(x^3)$. Write out the first six terms. Now find the Maclaurin series for $f(x) = \int \cos(x^3)dx$. Write out the first 4 terms. Use this to estimate $\int_0^1 \cos(x^3)\,dx$

Answer: $e^{-x^2} = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{n!}$, the intgral is $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)n!}$. Write out the first 6 terms. Then treat it like a polynomial and evaluate at x = 1 and x = 0 the limits of integration. Tis gives a good approximation of the definite integral

Find an equation for the line tangent to the curve at the point defined by the given value of t.

11) $x = t + \cos t$, $y = 2 - \sin t$, $t = \frac{\pi}{6}$

Answer: $y = -\sqrt{3}x + \frac{\sqrt{3}}{6}\pi + 3$

12) $x = 7t^2 - 4$, $y = t^3$, $t = 1$

Answer: $y = \frac{3}{14}x + \frac{5}{14}$

13) $x = \sqrt{t+3}$, $y = -t$, $t = 1$

Answer: $y = -4x - 5$

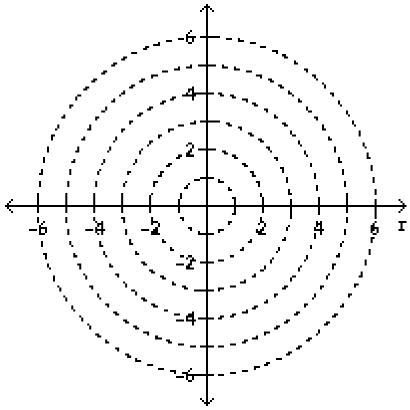
Find the value of $\frac{d^2y}{dx^2}$ for the curve at the point defined by the given value of t.

14) $x = t$, $y = \sqrt{2t}$, $t = 1$

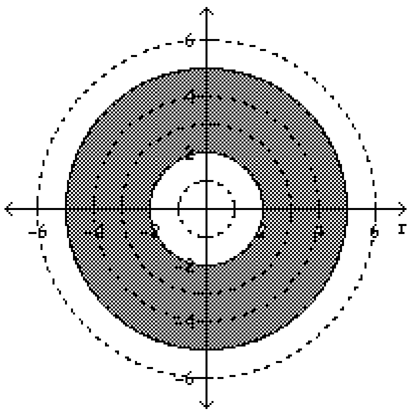
Answer: $-\frac{1}{2\sqrt{2}}$

Graph the set of points whose polar coordinates satisfy the given equation or inequality.

15) $2 \le r \le 5$



Answer:



Replace the polar equation with an equivalent Cartesian equation.

16) $r = 10 \sin \theta$

Answer: $x^2 + y^2 = 10y$

Find the slope of the polar curve at the indicated point.

17) $r = 9 \cos 3\theta, \theta = \frac{5\pi}{6}$

Answer: $-\frac{\sqrt{3}}{3}$ Find $x = r \cos\theta$ and $y = r\sin\theta$. From there find $dy/d\theta$ and divide by $dx/d\theta$.

Solve the problem.

18) Find values of θ for the horizontal and vertical tangent lines to the curve $r = 1 - \cos \theta$, $0 \le \theta < 2\pi$.

Answer: Horizontal: $\theta = 0, \frac{2\pi}{3}, \frac{4\pi}{3}$ vertical: $\theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$.

Find the area of the specified region.

19) Inside one leaf of the 4-leaved rose $r = 9 \sin 2\theta$

Answer: $\frac{81\pi}{8}$

20) Shared by the circle $r = 5$ and the cardioid $r = 5(1 + \sin \theta)$

Answer: $\frac{25}{4}(5\pi - 8)$

Find the length of the curve.

21) The cardioid $r = 7(1 - \cos \theta)$

Answer: 56

22) The spiral $r = 5\theta^2, 0 \le \theta \le 2\sqrt{3}$

Answer: $\frac{280}{3}$