Math 260
Summer 2022 Name:
Exam 3

1. 10 pts. Given vector spaces V and W, let $T : \mathsf{V} \to \mathsf{W}$ be a linear transformation. If U is a subspace of V, show that the set

$$T(\mathsf{U}) = \{T(\mathbf{x}) : \mathbf{x} \in U\}$$

(called the **image of U under T**) is a subspace of W.

2. 10 pts. Find a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}.$$

3. 10 pts. Find a basis for the space spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}.$$

4. 10 pts. Show that

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$

is a basis for $\mathbb{R}^2$, then find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis for $\mathbb{R}^2$.

5. 10 pts. Let U be an n-dimensional subspace of an n-dimensional vector space V. Prove that $\mathsf{U} = \mathsf{V}$.

6. 15 pts. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$, where

$$\mathbf{b}_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, and also the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

7. 15 pts. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis

$$\mathcal{B} = \{1 - 2t + t^2, 3 - 5t + 4t^2, 2t + 3t^2\}$$

to the standard basis $\mathcal{C} = \{1, t, t^2\}$. Then find the $\mathcal{B}$-coordinate vector for $t - t^2$.

8. 10 pts. Show that λ is an eigenvalue of A if and only if λ is an eigenvalue of $A^\top$.

9. 10 pts. Find the characteristic polynomial and eigenvalues of

$$A = \begin{bmatrix} 7 & -2 \\ 2 & 3 \end{bmatrix}.$$

10. 15 pts. If possible, diagonalize the matrix

$$A = \begin{bmatrix} 4 & 0 & 2 \\ 2 & 3 & 4 \\ 0 & 0 & 3 \end{bmatrix}$$

whose eigenvalues of 3 and 4.