MATH 260
SUMMER 2022 NAME:
EXAM 2

1. [10 pts.] Let I_n be the $n \times n$ identity matrix. Show that $AI_n = A$ for any $m \times n$ matrix A.

2. [10 pts.] Find the inverse of

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 4 & -7 & 3 \\ -2 & 6 & -4 \end{bmatrix},$$

or show that A is invertible.

3. [10 pts.] The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by $T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$ is linear. Show that T is invertible and find a formula for T^{-1}.

4. [10 pts.] If I is the 2×2 identity matrix and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, determine what conditions the entries of A must satisfy (if any) to ensure that $\det(A + I) = \det A + \det I$.

5. [10 pts.] Show that if C is a square matrix such that $C^\top C = I$, then $|\det C| = 1$.

6. [10 pts.] Use Cramer's Rule to solve the system

$$\begin{cases} -5x_1 + 2x_2 = 9 \\ 3x_1 - x_2 = -4 \end{cases}$$

7. [10 pts.] Let $\mathbb{R}^{m\times n}$ denote the set of all $m \times n$ matrices with real-valued entries. Given a matrix $C \in \mathbb{R}^{3\times 2}$, let H be the set of all $X \in \mathbb{R}^{2\times 4}$ such that $CX = O$ (the 3×4 zero matrix). Prove or disprove that H is a subspace of $\mathbb{R}^{2\times 4}$.

8. [10 pts.] The set of all continuous real-valued functions defined on a closed interval $[a, b]$ in $\mathbb{R}$ is denoted by $\mathcal{C}[a, b]$ and is a vector space. Show that the set

$$\mathsf{H} = \{f \in \mathcal{C}[a, b] : f(a) = f(b)\}$$

is a subspace of $\mathcal{C}[a, b]$

9. [10 pts.] Let H and K be subspaces of vector space V. Define the **sum** of H and K to be the set

$$\mathsf{H} + \mathsf{K} = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in \mathsf{H} \text{ and } \mathbf{v} \in \mathsf{K}\}.$$

Show that $\mathsf{H} + \mathsf{K}$ is a subspace of V.