

1. 10 pts. Determine the values of g and h such that the system

$$\begin{cases} x_1 + 3x_2 = g \\ 4x_1 + hx_2 = 8 \end{cases}$$

has (a) no solution; (b) a unique solution; and (c) many solutions.

2. 10 pts. Find the general solution to the system having augmented matrix

$$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix}$$

3. 10 pts. Determine if \mathbf{b} is a linear combination of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 :

$$\mathbf{a}_1 = \begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \quad \mathbf{a}_2 = \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \quad \mathbf{a}_3 = \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}.$$

4. 10 pts. Write the augmented matrix for the linear system that corresponds to the matrix equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

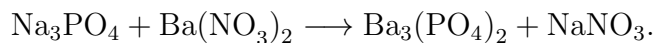
then solve the system and write the solution as a vector.

5. 10 pts. Given

$$\mathbf{v}_1 = \begin{bmatrix} 0 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 0 \\ -3 \\ 8 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 4 \\ -1 \\ -5 \end{bmatrix},$$

prove or disprove that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ spans \mathbb{R}^3 .

6. 10 pts. When solutions of sodium phosphate and barium nitrate are mixed, the result is barium phosphate and sodium nitrate. The unbalanced equation is



Use the vector equation approach to balance the equation.

7. 10 pts. Find the values of h for which the vectors are linearly dependent:

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \quad \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}.$$

8. 10 pts. Let $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, $\mathbf{v}_1 = \begin{bmatrix} -2 \\ 5 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$, and let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ be a linear transformation that maps \mathbf{x} into $x_1\mathbf{v}_1 + x_2\mathbf{v}_2$. Find a matrix A such that $T(\mathbf{x}) = A\mathbf{x}$ for each \mathbf{x} .
9. 10 pts. An **affine transformation** $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ has the form $T(\mathbf{x}) = A\mathbf{x} + \mathbf{b}$, with $A \in \mathbb{R}^{m \times n}$ (i.e. A is an $m \times n$ matrix) and $\mathbf{b} \in \mathbb{R}^m$. Show that T is not a linear transformation when $\mathbf{b} \neq \mathbf{0}$.
10. 10 pts. each Let $T : \mathbb{R}^3 \rightarrow \mathbb{R}^2$ be a linear transformation such that $T(\mathbf{e}_1) = (1, 3)$, $T(\mathbf{e}_2) = (4, -7)$, and $T(\mathbf{e}_3) = (-5, 4)$.
- (a) Prove or disprove that T one-to-one.
- (b) Prove or disprove that T onto.
11. 10 pts. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation such that $T(\mathbf{x}) = B\mathbf{x}$ for some $m \times n$ matrix B . Show that if A is the standard matrix for T , then $A = B$.

12. 10 pts. Let

$$A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}.$$

Find the values of k , if any, for which $AB = BA$.

Theorem 1: Suppose equation $A\mathbf{x} = \mathbf{b}$ is consistent for some vector \mathbf{b} , and let \mathbf{p} be a particular solution. If S and S_h are the solution sets for $A\mathbf{x} = \mathbf{b}$ and $A\mathbf{x} = \mathbf{0}$, respectively, then $S = \{\mathbf{p} + \mathbf{v}_h : \mathbf{v}_h \in S_h\}$.

Theorem 2: Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation, and let A be the standard matrix for T . Then:

- a) T is onto if and only if the columns of A span \mathbb{R}^m .
- b) T is one-to-one if and only if the columns of A are linearly independent.