MATH 260
SUMMER 2021
EXAM 2

NAME:

1. [10 pts.] If I is the 2×2 identity matrix and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, determine what conditions the entries of A must satisfy (if any) to ensure that $\det(A + I) = \det A + \det I$.

2. [10 pts.] Use Cramer's Rule to solve the system

$$\begin{cases} x_1 + x_2 = 3 \\ -3x_1 + 2x_3 = 0 \\ x_2 - 2x_3 = 2 \end{cases}$$

3. [10 pts.] Let W be the set of all vectors of the form

$$\begin{bmatrix} s + 3t \\ s - t \\ 2s + 6t \end{bmatrix}.$$

Prove or disprove that W is a subspace of $\mathbb{R}^3$.

4. [10 pts.] The set of all continuous real-valued functions defined on a closed interval $[a, b]$ in $\mathbb{R}$ is denoted by $\mathcal{C}[a, b]$ and is a vector space. Show that the set $\{f \in \mathcal{C}[a, b] : f(a) = f(b)\}$ is a subspace of $\mathcal{C}[a, b]$.

5. [10 pts.] Let H and K be subspaces of vector space V. Define the **sum** of H and K to be the set

$$H + K = \{\mathbf{u} + \mathbf{v} : \mathbf{u} \in H \text{ and } \mathbf{v} \in K\}.$$

Show that $H + K$ is a subspace of V.

6. [10 pts.] Find a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}.$$

7. [10 pts.] Find a basis for the space spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}.$$

8. 15 pts. Show that

$$\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \end{bmatrix} \right\}$$

is a basis for $\mathbb{R}^2$, then find the change-of-coordinates matrix from $\mathcal{B}$ to the standard basis for $\mathbb{R}^2$.

9. 10 pts. The set of polynomials $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$ relative to $\mathcal{B}$.

10. 10 pts. Show that the space $\mathcal{C}(\mathbb{R})$ of all continuous functions defined on the real line is an infinite-dimensional space.

11. 15 pts. The matrices

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 1 & -1 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

are row equivalent. What is rank A and dim Nul A? Also find bases for Col A, Row A, and Nul A.

12. 15 pts. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$, where

$$\mathbf{b}_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, and also the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.

13. 10 pts. In $\mathbb{P}_2$, find the change-of-coordinates matrix from the basis $\mathcal{B} = \{1 - 3t^2, 2 + t - 5t^2, 1 + 2t\}$ to the standard basis $\mathcal{C} = \{1, t, t^2\}$.