Math 260
Summer 2020
Exam 3

Name:

1. [10 pts.] For the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & -1 \\ 1 & -3 & 0 \\ 4 & -13 & 1 \end{bmatrix},$$

find a basis for the eigenspace corresponding to the eigenvalue $\lambda = -2$.

2. [10 pts.] Suppose $\mathbf{B}$ is an $m \times m$ matrix whose entries along each row add up to the same value μ. Show that μ is an eigenvalue of $\mathbf{B}$.

3. [10 pts.] Find the characteristic equation of

$$\mathbf{A} = \begin{bmatrix} -2 & 0 & 3 \\ 5 & -2 & 8 \\ 1 & -4 & 6 \end{bmatrix}.$$

4. [10 pts.] Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = \mathbf{A}\mathbf{x}$, where

$$\mathbf{A} = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}.$$

Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ such that $[T]_{\mathcal{B}}$ is diagonal.

5. [10 pts.] Find the distance from $\mathbf{x}$ to the line through $\mathbf{u}$ and the origin, where

$$\mathbf{x} = \begin{bmatrix} -1 \\ -6 \end{bmatrix} \quad \text{and} \quad \mathbf{u} = \begin{bmatrix} -8 \\ 3 \end{bmatrix}.$$

6. [10 pts.] For $\mathbf{u} \in \mathbb{R}^n$ such that $\mathbf{u} \neq \mathbf{0}$, define $\ell = \text{Span}\{\mathbf{u}\}$. Show that the mapping $\mathbf{x} \mapsto 2(\text{proj}_\ell\, \mathbf{x}) - \mathbf{x}$ is a linear transformation.

7. [10 pts.] Given

$$\mathbf{x} = \begin{bmatrix} -3 \\ 8 \\ 2 \\ -4 \end{bmatrix}, \quad \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} 0 \\ -1 \\ 1 \\ -1 \end{bmatrix},$$

let $W = \text{Span}\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$. Express $\mathbf{x}$ as a sum $\mathbf{v} + \mathbf{w}$, where $\mathbf{v} \in W$ and $\mathbf{w} \in W^\perp$.

8. [10 pts.] Find an orthogonal basis for Col $\mathbf{A}$, where

$$\mathbf{A} = \begin{bmatrix} 3 & -8 & 3 \\ 1 & -4 & -3 \\ -1 & 6 & 6 \\ 1 & -2 & 6 \end{bmatrix}.$$

9. [10 pts.] Suppose $\mathbb{P}_3$ has inner product given by

$$\langle p, q \rangle = p(-3)q(-3) + p(-1)q(-1) + p(1)q(1) + p(3)q(3).$$

Define polynomials $p_0(t) = 1$, $p_1(t) = t$, $p_2(t) = \frac{1}{4}t^2 - \frac{5}{4}$. Determine the best approximation to $r(t) = t^3$ by polynomials in $\text{Span}\{p_0, p_1, p_2\}$.

10. [10 pts.] Let V be a vector space. Suppose $T : V \to \mathbb{R}^n$ is a one-to-one linear transformation. Prove that $\langle \mathbf{v}_1, \mathbf{v}_2 \rangle = T(\mathbf{v}_1) \cdot T(\mathbf{v}_2)$ is an inner product on V.

11. [10 pts.] Recall $\mathcal{C}[a, b]$ denotes the vector space of real-valued functions that are continuous on the interval $[a, b]$. For $f, g \in \mathcal{C}[a, b]$ define

$$\langle f, g \rangle = \int_a^b f(t)g(t)\,dt,$$

which is an inner product on $\mathcal{C}[a, b]$. Find an orthogonal basis for the subspace of $\mathcal{C}[-3, 3]$ that is spanned by polynomials 3, $2t$, t^2.