MATH 260
SUMMER 2020
EXAM 2

NAME:

1. [10 pts.] Given that

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = -9,$$

evaluate the determinant

$$\begin{vmatrix} -3g & -3h & -3i \\ d+4g & e+4h & f+4i \\ -2a & -2b & -2c \end{vmatrix}.$$

2. [10 pts.] Suppose $\mathbf{A}$ is a square matrix such that $\det(\mathbf{A}^7) = 0$. Explain why $\mathbf{A}$ cannot be invertible.

3. [10 pts.] Use Cramer's Rule to solve the system

$$\begin{cases} x + 3y + z = 4 \\ -x + 2z = 2 \\ 3x + y = 2 \end{cases}$$

4. [10 pts.] Find the volume of the parallelepiped with one vertex at the origin and adjacent vertices at $(1, 3, 0)$, $(-2, 0, 2)$, and $(-1, 3, -1)$.

5. [10 pts.] Let W be the set of all vectors of the form

$$\begin{bmatrix} s+3t \\ s-t \\ 2s-7t \\ 4t \end{bmatrix}.$$

Prove or disprove that W is a subspace of $\mathbb{R}^4$.

6. [10 pts.] The set of all continuous real-valued functions defined on a closed interval $[a, b]$ in $\mathbb{R}$ is denoted by $\mathcal{C}[a, b]$ and is a vector space. Show that the set $\{f \in \mathcal{C}[a, b] : f(a) = f(b)\}$ is a subspace of $\mathcal{C}[a, b]$. The symbol $\in$ means "is an element of," as usual.

7. [10 pts.] Prove or disprove that the set

$$W = \left\{ \begin{bmatrix} r \\ s \\ t \end{bmatrix} : 6r - 2 = s - 9t \right\}$$

is a vector space.

8. 10 pts. Find a basis for the null space of the matrix

$$\begin{bmatrix} 1 & 0 & -5 & 1 & 4 \\ -2 & 1 & 6 & -2 & -2 \\ 0 & 2 & -8 & 1 & 9 \end{bmatrix}.$$

9. 10 pts. Find a basis for the space spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ -1 \\ 1 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 6 \\ -1 \\ 2 \\ -1 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 5 \\ -3 \\ 3 \\ -4 \end{bmatrix}, \quad \mathbf{v}_5 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 1 \end{bmatrix}.$$

10. 10 pts. Find the change-of-coordinates matrix from the basis

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ -1 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}, \begin{bmatrix} 8 \\ -2 \\ 7 \end{bmatrix} \right\}$$

to the standard basis for $\mathbb{R}^3$.

11. 10 pts. The set of polynomials $\mathcal{B} = \{1 - t^2, t - t^2, 2 - 2t + t^2\}$ is a basis for $\mathbb{P}_2$. Find the coordinate vector of $\mathbf{p}(t) = 3 + t - 6t^2$ relative to $\mathcal{B}$.

12. 10 pts. Determine the dimensions of Nul $\mathbf{A}$ and Col $\mathbf{A}$ for the matrix

$$\mathbf{A} = \begin{bmatrix} 1 & 3 & -4 & 2 & -1 & 6 \\ 0 & 0 & 1 & -3 & 7 & 0 \\ 0 & 0 & 0 & 1 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

13. 10 pts. Show that the space $C(\mathbb{R})$ of all continuous functions defined on the real line is an infinite-dimensional space.

14. 10 pts. In statistical theory, a common requirement is that a matrix be of **full rank**. That is, the rank should be as large as possible. Explain why an $m \times n$ matrix with more rows than columns has full rank if and only if its columns are linearly independent.

15. 10 pts. Let $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$ and $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2\}$ be bases for $\mathbb{R}^2$, where

$$\mathbf{b}_1 = \begin{bmatrix} -1 \\ 8 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \quad \mathbf{c}_1 = \begin{bmatrix} 1 \\ 4 \end{bmatrix}, \quad \mathbf{c}_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Find the change-of-coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$, and the change-of-coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$.