

1. [5 pts.] Determine the values of h such that the matrix

$$\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix}$$

is the augmented matrix of a consistent linear system.

2. [10 pts.] Find the general solution to the system having augmented matrix

$$\begin{bmatrix} 0 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix}.$$

3. [10 pts.] Determine if \mathbf{b} is a linear combination of the column vectors of the matrix \mathbf{A} :

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.$$

4. [10 pts.] For what values of h is the vector \mathbf{y} in the plane generated by \mathbf{v}_1 and \mathbf{v}_2 , given that

$$\mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}?$$

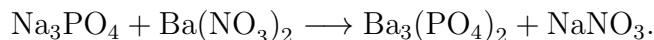
5. [10 pts.] Write the augmented matrix for the linear system that corresponds to the matrix equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

then solve the system and write the solution as a vector.

6. [10 pts.] Suppose $\mathbf{Ax} = \mathbf{b}$ has a solution. Show that the solution is unique if and only if $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution.

7. [10 pts.] When solutions of sodium phosphate and barium nitrate are mixed, the result is barium phosphate and sodium nitrate. The unbalanced equation is



Use the vector equation approach to balance the equation.

8. 10 pts. Prove or disprove the following statement: If $\mathbf{v}_1, \dots, \mathbf{v}_4$ are in \mathbb{R}^4 and \mathbf{v}_3 is *not* a linear combination of $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_4$, then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.
9. 10 pts. Show that the transformation T defined by $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ is not linear.

10. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2).$$

- (a) 10 pts. Find $\mathbf{x} \in \mathbb{R}^2$ such that $T(\mathbf{x}) = (-1, 4, 9)$.
- (b) 10 pts. Prove or disprove that the transformation T one-to-one.
- (c) 10 pts. Prove or disprove that the transformation T onto.
11. 10 pts. Let

$$\mathbf{A} = \begin{bmatrix} 3 & -4 \\ -1 & 8 \end{bmatrix}.$$

Either construct a 2×2 matrix \mathbf{B} having two different nonzero columns such that \mathbf{AB} is the zero matrix, or show such a matrix \mathbf{B} cannot exist.

12. 10 pts. Suppose $(\mathbf{B} - \mathbf{C})\mathbf{D} = \mathbf{O}$, where \mathbf{B} and \mathbf{C} are $m \times n$ matrices and \mathbf{D} is invertible. Show that $\mathbf{B} = \mathbf{C}$.
13. 15 pts. Show that the linear transformation $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ given by

$$T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$$

is invertible, and find a formula for T^{-1} .

14. 10 pts. Prove or disprove that the vectors

$$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$$

form a basis for \mathbb{R}^3 .

15. 10 pts. Let

$$\mathbf{b}_1 = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}.$$

The vector \mathbf{x} is in a subspace H with basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the \mathcal{B} -coordinates of \mathbf{x} .

16. 10 pts. What is the rank of a 4×6 matrix whose null space is two dimensional? Justify your answer.