Math 260
Summer 2020 Name:
Exam 1

1. $\boxed{\text{5 pts.}}$ Determine the values of h such that the matrix

$$\begin{bmatrix} 2 & -3 & h \\ -6 & 9 & 5 \end{bmatrix}$$

is the augmented matrix of a consistent linear system.

2. $\boxed{\text{10 pts.}}$ Find the general solution to the system having augmented matrix

$$\begin{bmatrix} 0 & -2 & -1 & 3 \\ 3 & -6 & -2 & 2 \end{bmatrix}.$$

3. $\boxed{\text{10 pts.}}$ Determine if $\mathbf{b}$ is a linear combination of the column vectors of the matrix $\mathbf{A}$:

$$\mathbf{A} = \begin{bmatrix} 1 & -2 & -6 \\ 0 & 3 & 7 \\ 1 & -2 & 5 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 11 \\ -5 \\ 9 \end{bmatrix}.$$

4. $\boxed{\text{10 pts.}}$ For what values of h is the vector $\mathbf{y}$ in the plane generated by $\mathbf{v}_1$ and $\mathbf{v}_2$, given that

$$\mathbf{y} = \begin{bmatrix} h \\ -5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -3 \\ 1 \\ 8 \end{bmatrix}?$$

5. $\boxed{\text{10 pts.}}$ Write the augmented matrix for the linear system that corresponds to the matrix equation $\mathbf{Ax} = \mathbf{b}$, where

$$\mathbf{A} = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

then solve the system and write the solution as a vector.

6. $\boxed{\text{10 pts.}}$ Suppose $\mathbf{Ax} = \mathbf{b}$ has a solution. Show that the solution is unique if and only if $\mathbf{Ax} = \mathbf{0}$ has only the trivial solution.

7. $\boxed{\text{10 pts.}}$ When solutions of sodium phosphate and barium nitrate are mixed, the result is barium phosphate and sodium nitrate. The unbalanced equation is

$$\mathrm{Na_3PO_4} + \mathrm{Ba(NO_3)_2} \longrightarrow \mathrm{Ba_3(PO_4)_2} + \mathrm{NaNO_3}.$$

Use the vector equation approach to balance the equation.

8. $\boxed{\text{10 pts.}}$ Prove or disprove the following statement: If $\mathbf{v}_1, \ldots, \mathbf{v}_4$ are in $\mathbb{R}^4$ and $\mathbf{v}_3$ is *not* a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_4$, then the set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is linearly independent.

9. $\boxed{\text{10 pts.}}$ Show that the transformation T defined by $T(x_1, x_2) = (4x_1 - 2x_2, 3|x_2|)$ is not linear.

10. Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a linear transformation such that

$$T(x_1, x_2) = (x_1 - 2x_2, -x_1 + 3x_2, 3x_1 - 2x_2).$$

   (a) $\boxed{\text{10 pts.}}$ Find $\mathbf{x} \in \mathbb{R}^2$ such that $T(\mathbf{x}) = (-1, 4, 9)$.

   (b) $\boxed{\text{10 pts.}}$ Prove or disprove that the transformation T one-to-one.

   (c) $\boxed{\text{10 pts.}}$ Prove or disprove that the transformation T onto.

11. $\boxed{\text{10 pts.}}$ Let

$$\mathbf{A} = \begin{bmatrix} 3 & -4 \\ -1 & 8 \end{bmatrix}.$$

Either construct a 2×2 matrix $\mathbf{B}$ having two different nonzero columns such that $\mathbf{AB}$ is the zero matrix, or show such a matrix $\mathbf{B}$ cannot exist.

12. $\boxed{\text{10 pts.}}$ Suppose $(\mathbf{B} - \mathbf{C})\mathbf{D} = \mathbf{O}$, where $\mathbf{B}$ and $\mathbf{C}$ are $m \times n$ matrices and $\mathbf{D}$ is invertible. Show that $\mathbf{B} = \mathbf{C}$.

13. $\boxed{\text{15 pts.}}$ Show that the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by

$$T(x_1, x_2) = (6x_1 - 8x_2, -5x_1 + 7x_2)$$

is invertible, and find a formula for T^{-1}.

14. $\boxed{\text{10 pts.}}$ Prove or disprove that the vectors

$$\begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \quad \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \quad \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$$

form a basis for $\mathbb{R}^3$.

15. $\boxed{\text{10 pts.}}$ Let

$$\mathbf{b}_1 = \begin{bmatrix} -3 \\ 1 \\ -4 \end{bmatrix}, \quad \mathbf{b}_2 = \begin{bmatrix} 7 \\ 5 \\ -6 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} 11 \\ 0 \\ 7 \end{bmatrix}.$$

The vector $\mathbf{x}$ is in a subspace H with basis $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2\}$. Find the $\mathcal{B}$-coordinates of $\mathbf{x}$.

16. $\boxed{\text{10 pts.}}$ What is the rank of a 4×6 matrix whose null space is two dimensional? Justify your answer.