Math 260
Summer 2018
Exam 2

Name:

1. [10 pts.] Prove that the intersection of two convex sets in a vector space V is convex.

2. [10 pts.] The set $\mathcal{B} = \{[-6,1], [-3,2]\}$ is a basis for $\mathbb{R}^2$. Find the coordinates of $\mathbf{x} = [4,-8]$ with respect to $\mathcal{B}$.

3. [10 pts.] Consider the vector space of functions defined on the interval $(0,\infty)$. Prove or disprove that the set of functions $\{e^t, \ln t\}$ is linearly independent.

4. [10 pts.] Find the rank of the matrix, justifying your answer:

$$\mathbf{M} = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 9 & -2 & 1 & 0 \\ -1 & 0 & -1 & 8 \\ 4 & -8 & 2 & -12 \end{bmatrix}.$$

5. [10 pts.] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear mapping such that $L(1,1) = (2,-1)$ and $L(-1,3) = (1,2)$. Find $L(0,1)$.

6. [10 pts.] Let V, W be vector spaces and $L : V \to W$ a linear mapping. Suppose $\mathbf{w}_1, \ldots, \mathbf{w}_n \in W$ are linearly independent and $\mathbf{v}_1, \ldots, \mathbf{v}_n \in V$ are such that $L(\mathbf{v}_k) = \mathbf{w}_k$ for $1 \le k \le n$. Show that $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent.

7. [10 pts.] Let $L : V \to W$ be a linear mapping. Assume $\dim V > \dim W$. Prove that the kernel of L is not $\{\mathbf{0}\}$.

8. [10 pts.] Find the dimension of the subspace of $\mathbb{R}^5$ orthogonal to the vectors $[1,1,-2,3,4]$, $[1,0,0,2,0]$, $[0,1,0,1,0]$.

9. [10 pts.] Find the matrix corresponding to the linear mapping $L : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$L\big([x_1, x_2, x_3, x_4]^\top\big) = [2x_3, 0, -2x_1]^\top$$

with respect to the standard bases.

10. [10 pts.] Let $L : \mathbb{R}^{1\times 2} \to \mathbb{R}^{1\times 3}$ be the linear transformation defined by

$$L(x,y) = [8x+y, 3x-5y, x-4y].$$

Prove or disprove that L is invertible.

11. 10 pts. The ordered sets

$$\mathcal{B} = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right)$$

are bases for $\mathbb{R}^2$. Find the change of basis matrix $\mathbf{I}_{\mathcal{BC}}$ (a.k.a. transition matrix) for changing from the basis $\mathcal{B}$ to the basis $\mathcal{C}$.