MATH 260
SUMMER 2018 NAME:
EXAM 1

1. $\boxed{\text{5 pts. each}}$ Let $\mathbf{u} = [3, -1, 4, 8]$.

   (a) Find a vector parallel to $\mathbf{u}$ that has length 2.

   (b) Find a vector of the form $[x, 6, y, -3]$ that is orthogonal to $\mathbf{u}$, if such exists.

2. $\boxed{\text{5 pts. each}}$ Let $\mathbf{u} = [5, 5, -2]$ and $\mathbf{v} = [-2, -1, 1]$.

   (a) Find $\text{proj}_{\mathbf{v}}\, \mathbf{u}$, the projection of $\mathbf{u}$ along $\mathbf{v}$.

   (b) Find $\text{proj}_{\mathbf{u}}\, \mathbf{v}$.

   (c) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

   (d) Show that, for any nonzero vectors $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$, the angle between $\text{proj}_{\mathbf{y}}\, \mathbf{x}$ and $\text{proj}_{\mathbf{x}}\, \mathbf{y}$ is always the same as the angle between $\mathbf{x}$ and $\mathbf{y}$.

3. $\boxed{\text{10 pts.}}$ Given points $p = (1, 2, -3)$ and $q = (8, 1, -1)$, find the point that lies one-third of the way from p to q on the line segment between p and q.

4. $\boxed{\text{10 pts.}}$ Find a parametric equation for the line passing through the points $p = (1, 0, -1)$ and $q = (2, 2, -3)$.

5. $\boxed{\text{10 pts.}}$ Find a parametric equation $\mathbf{x}(t)$ for the line of intersection of the two planes $x - 2y + z = 0$ and $2x - 3y + z = 6$.

6. $\boxed{\text{10 pts.}}$ Find $\mathbf{BAC}$ for

$$\mathbf{A} = \begin{bmatrix} 2 & a & 1 \\ 3 & -5 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 3 \\ 5 & -3 \\ a & -1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 9 \\ 1 \\ 2 \end{bmatrix}.$$

7. $\boxed{\text{10 pts.}}$ Suppose that $\mathbf{A}^3 - \mathbf{A} + \mathbf{I} = \mathbf{O}$. Show that $\mathbf{A}$ is invertible.

8. Let $\mathbf{A}$ and $\mathbf{B}$ be two $n \times n$ matrices. We say $\mathbf{A}$ is **similar** to $\mathbf{B}$ if there exists an invertible matrix $\mathbf{T}$ such that $\mathbf{B} = \mathbf{TAT}^{-1}$. Suppose this is the case. Prove the following.

   (a) $\boxed{\text{5 pts.}}$ $\mathbf{B}$ is similar to $\mathbf{A}$.

   (b) $\boxed{\text{10 pts.}}$ $\mathbf{A}$ is invertible if and only if $\mathbf{B}$ is invertible.

9. $\boxed{\text{10 pts.}}$ Find the inverse for the matrix using elementary row operations, if the inverse exists:

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

10. $\boxed{\text{10 pts.}}$ Find the solution set of the system using Gaussian elimination:

$$\begin{cases} x + 2y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

11. $\boxed{\text{10 pts.}}$ Find the complete solution set of the system:

$$\begin{cases} 3x - y + 6z = 4 \\ x + y - 2z = -1 \end{cases}$$

12. $\boxed{\text{10 pts.}}$ Show that the set of vectors $[x, y, z]$ in $\mathbb{R}^3$ that satisfy $x - 7y + 3z = 0$ is a subspace of $\mathbb{R}^3$, then find a basis for this subspace.

13. $\boxed{\text{10 pts.}}$ Prove or disprove that the set of vectors $[x, y]$ in $\mathbb{R}^2$ that satisfy $\sin x - 2y = 0$ is a subspace of $\mathbb{R}^2$.

14. $\boxed{\text{10 pts.}}$ Let $[a, b]$ and $[c, d]$ be two vectors in $\mathbb{R}^2$. Prove that they are linearly independent if $ad - bc \neq 0$.

15. $\boxed{\text{10 pts.}}$ Prove or disprove that the vectors $[1, 2, 0]$, $[1, 3, -1]$, $[-1, 1, 1]$ are linearly independent vectors in $\mathbb{R}^3$.