MATH 260
SUMMER 2017 NAME:
EXAM 1

Vector Space Axioms:
VS1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ for any $\mathbf{u},\mathbf{v}\in V$
VS2. $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$ for any $\mathbf{u},\mathbf{v},\mathbf{w}\in V$
VS3. There exists some $\mathbf{0}\in V$ such that $\mathbf{u}+\mathbf{0}=\mathbf{u}$ for any $\mathbf{u}\in V$
VS4. For each $\mathbf{u}\in V$ there exists some $-\mathbf{u}\in V$ such that $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$
VS5. For any $a\in\mathbb{F}$ and $\mathbf{u},\mathbf{v}\in V$, $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$
VS6. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $(a+b)\mathbf{u}=a\mathbf{u}+b\mathbf{u}$
VS7. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $a(b\mathbf{u})=(ab)\mathbf{u}$
VS8. For all $\mathbf{u}\in V$, $1\mathbf{u}=\mathbf{u}$

1. $\boxed{\text{5 pts. each}}$ Let $\mathbf{u}=[-3,9,2]$ and $\mathbf{v}=[-2,-1,0]$.
   (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   (b) Find $\text{proj}_{\mathbf{v}}\,\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ in exact terms.

2. $\boxed{\text{10 pts.}}$ Using properties of the dot product (a.k.a. scalar product), for vectors $\mathbf{u}$ and $\mathbf{v}$ rewrite the expression
$$\|\mathbf{u}+\mathbf{v}\|^2-\|\mathbf{u}-\mathbf{v}\|^2$$
in the form $c\mathbf{u}\cdot\mathbf{v}$ for some appropriate constant c.

3. $\boxed{\text{10 pts.}}$ Determine the coordinates of the point lying one-fifth of the way from $p=(1,3,-1)$ to $q=(-4,2,6)$ along the line segment $\overline{pq}$.

4. $\boxed{\text{10 pts.}}$ Find an equation of the plane in $\mathbb{R}^3$ that is perpendicular to $\mathbf{n}=[-3,2,8]$ and contains the point $p=(1/2,0,2)$.

5. $\boxed{\text{10 pts.}}$ Find a parametric equation $\mathbf{x}(t)$ for the line of intersection of the two planes $x-y+z=2$ and $2x-3y+z=1$.

6. $\boxed{\text{10 pts.}}$ Find $\mathbf{ABC}$ for
$$\mathbf{A}=\begin{bmatrix}2 & a & 1\\ 3 & -1 & 2\end{bmatrix},\quad \mathbf{B}=\begin{bmatrix}1 & 1\\ 2 & -3\\ a & -1\end{bmatrix},\quad \mathbf{C}=\begin{bmatrix}1\\ 3\end{bmatrix}.$$

7. $\boxed{\text{10 pts.}}$ Suppose that $\mathbf{A}^2+2\mathbf{A}+\mathbf{I}=\mathbf{O}$. Show that $\mathbf{A}$ is invertible.

8. $\boxed{\text{10 pts.}}$ Find the inverse for the matrix using elementary row operations, if the inverse exists:

$$\mathbf{C} = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$$

9. $\boxed{\text{10 pts.}}$ Solve the system using Gaussian elimination:

$$\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$$

10. $\boxed{\text{10 pts.}}$ Find the complete solution set of the system:

$$\begin{cases} 3x - y + 6z = 4 \\ x + y - 2z = 0 \end{cases}$$

11. $\boxed{\text{10 pts.}}$ Prove or disprove that the set

$$S = \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| \, x^2 + y^2 \le 9 \right\}$$

is a vector space under the usual vector operations for $\mathbb{R}^2$.

12. $\boxed{\text{10 pts.}}$ The sets $U = \{[x, y] : 4y - 3x = 0\}$ and $V = \{[x, y] : y + 8x = 0\}$ are vector subspaces of $\mathbb{R}^2$. Show that $U \cup V$ is not a vector subspace.

13. $\boxed{\text{10 pts.}}$ Prove that if U_1 and U_2 are subspaces of a vector space V, then $U_1 + U_2$ is a subspace of V.

14. $\boxed{\text{10 pts.}}$ Prove or disprove that the vectors $[1, 2]$, $[1, 3]$ are linearly independent vectors in $\mathbb{R}^2$.