MATH 260
SUMMER 2016
EXAM 3

NAME:

1. An $n \times n$ symmetric matrix $\mathbf{A}$ with real entries is called positive definite if $\mathbf{x}^\top \mathbf{A}\mathbf{x} > 0$ for all $\mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{x} \neq \mathbf{0}$.

   (a) $\boxed{\text{5 pts.}}$ Show that
   $$\mathbf{A} = \begin{bmatrix} 4 & -1 \\ -1 & 10 \end{bmatrix}$$
   is positive definite

   (b) $\boxed{\text{10 pts.}}$ Let $a > 0$. Prove that
   $$\mathbf{A} = \begin{bmatrix} a & b \\ b & c \end{bmatrix}$$
   is positive definite if and only if $ac - b^2 > 0$.

2. $\boxed{\text{15 pts.}}$ Let V be the vector space of all continuous functions $f : [0,1] \to \mathbb{R}$, and define a scalar product on V by
$$\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt$$
for all $f, g \in V$. Let U be the subspace of V generated by $\{1, t, t^2\}$; that is, $U = \operatorname{Span}\{1, t, t^2\}$. Find an orthonormal basis for U.

3. $\boxed{\text{10 pts.}}$ Evaluate the determinant, which will be a polynomial in t:
$$\begin{vmatrix} -2 & 2 & 3 & -4 \\ 1 & 1 & t & 3 \\ -1 & 0 & 1 & -1 \\ -2 & 2 & 6 & -5 \end{vmatrix},$$

4. $\boxed{\text{10 pts.}}$ Solve the system using Cramer's Rule.
$$\begin{cases} 3x + y - z = 0 \\ x + y + z = 0 \\ y - z = 1 \end{cases}$$

5. Let
$$\mathbf{B} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}.$$

   (a) $\boxed{\text{5 pts.}}$ Find the characteristic equation for $\mathbf{B}$.

   (b) $\boxed{\text{5 pts.}}$ Find all real eigenvalues for $\mathbf{B}$.

   (c) $\boxed{\text{10 pts.}}$ Find a basis for the eigenspace corresponding to each real eigenvalue.

6. $\boxed{\text{5 pts. each}}$ The matrix

$$\mathbf{M} = \begin{bmatrix} 7 & -15 \\ 2 & -4 \end{bmatrix}$$

is diagonalizable.

(a) Find the characteristic equation for $\mathbf{M}$, and use it to find the eigenvalues of $\mathbf{M}$.

(b) For each eigenvalue of $\mathbf{M}$ find the basis for the corresponding eigenspace.

(c) Find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{M} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

(d) Find $\mathbf{M}^{12}$.

(e) Find $\mathbf{M}^{1/2}$.