MATH 260
SUMMER 2016 NAME:
EXAM 1

Vector Space Axioms:

VS1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ for any $\mathbf{u},\mathbf{v}\in V$

VS2. $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$ for any $\mathbf{u},\mathbf{v},\mathbf{w}\in V$

VS3. There exists some $\mathbf{0}\in V$ such that $\mathbf{u}+\mathbf{0}=\mathbf{u}$ for any $\mathbf{u}\in V$

VS4. For each $\mathbf{u}\in V$ there exists some $-\mathbf{u}\in V$ such that $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$

VS5. For any $a\in\mathbb{F}$ and $\mathbf{u},\mathbf{v}\in V$, $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$

VS6. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $(a+b)\mathbf{u}=a\mathbf{u}+b\mathbf{u}$

VS7. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $a(b\mathbf{u})=(ab)\mathbf{u}$

VS8. For all $\mathbf{u}\in V$, $1\mathbf{u}=\mathbf{u}$

1. [5 pts. each] Let $\mathbf{u}=[2,-1,4]$ and $\mathbf{v}=[-3,1,2]$.

   (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.

   (b) Find $\operatorname{proj}_{\mathbf{v}}\mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.

   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

2. [10 pts.] Find a parametrization $\mathbf{x}(t)$ of the line containing the points $p=(2,-6,9)$ and $q=(0,8,1)$, such that $\mathbf{x}(0)=\mathbf{p}$ and $\mathbf{x}(1)=\mathbf{q}$.

3. [10 pts.] Find an equation of the plane in $\mathbb{R}^3$ containing the points $(0,1,2)$, $(2,3,4)$, and $(4,5,6)$.

4. [10 pts.] Find a parametric equation $\mathbf{x}(t)$ for the line of intersection of the two planes $x-y+z=3$ and $2x-3y-z=1$.

5. [10 pts.] Find $\mathbf{BA}$ for

$$\mathbf{A}=\begin{bmatrix}2 & -3 & 0\\ -1 & 4 & 6\end{bmatrix} \quad\text{and}\quad \mathbf{B}=\begin{bmatrix}-7 & 5\\ 0 & 1\\ 3 & -9\end{bmatrix}.$$

6. Let $\mathbf{I}$ denote the 2×2 identity matrix, and $\mathbf{O}$ the 2×2 zero matrix.

   (a) [5 pts.] Find a 2×2 matrix $\mathbf{A}$ such that $\mathbf{A}^2=-\mathbf{I}$.

   (b) [10 pts.] Determine all 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2=\mathbf{O}$.

7. [15 pts.] Using elementary row operations, find a row-equivalent matrix for

$$\begin{bmatrix}0 & 1 & 3 & -2\\ 2 & 1 & -4 & 3\\ 2 & 3 & 2 & -1\end{bmatrix}$$

that is in row echelon form.

8. [15 pts.] Find the inverse for the matrix using elementary row operations, if the inverse exists:

$$\mathbf{C} = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 2 & 1 \\ 2 & 4 & 3 \end{bmatrix}$$

9. [15 pts.] Solve the system using Gaussian elimination:

$$\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$$

10. [15 pts.] Find the complete solution set of the system:

$$\begin{cases} 3x - 5y + 6z = 4 \\ x + y - 2z = -1 \end{cases}$$

11. [10 pts.] Show that the set

$$S = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x - 2y + z^2 = 0 \right\}$$

is not a vector space under the usual vector operations for $\mathbb{R}^3$.

12. [10 pts.] Show that the set $S = \{[x, y] : 4y - 3x = 0\}$ is a vector subspace of $\mathbb{R}^2$.

13. [10 pts.] Prove that if U and W are subspaces of a vector space V, then $U \cap W$ is a subspace of V.

14. [10 pts.] Let U be a subspace of $\mathbb{R}^n$. Prove that

$$U^{\perp} = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \cdot \mathbf{u} = 0 \text{ for all } \mathbf{u} \in U\}.$$

is also a subspace of $\mathbb{R}^n$.

15. [10 pts.] Prove or disprove that the vectors $[1, 2, 6]$, $[1, 5, -1]$, and $[0, 3, 1]$ are linearly independent.