MATH 260
SUMMER 2015 NAME:
EXAM 3

1. $\boxed{\text{10 pts.}}$ Evaluate the determinant

$$\begin{vmatrix} -1 & -3 & 2 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{vmatrix}.$$

2. $\boxed{\text{15 pts.}}$ Solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using Cramer's Rule, if possible, where

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 0 & 4 \\ 0 & 3 & 1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}.$$

3. $\boxed{\text{10 pts. each}}$ The matrix

$$\mathbf{A} = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix}$$

is diagonalizable.

(a) Find the characteristic polynomial of $\mathbf{A}$, and use it to find the eigenvalues of $\mathbf{A}$.

(b) For each eigenvalue of $\mathbf{A}$ find the basis for the corresponding eigenspace.

(c) Find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A} = \mathbf{P}\mathbf{D}\mathbf{P}^{-1}$.

(d) Find $\mathbf{A}^{50}$ and $\mathbf{A}^{1/2}$.

4. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}.$$

(a) $\boxed{\text{15 pts.}}$ Beginning with the vector $\mathbf{v}_1$, use the Gram-Schmidt Orthogonalization Process to obtain an orthogonal basis for W.

(b) $\boxed{\text{5 pts.}}$ Find an orthonormal basis for W.