MATH 260
SUMMER 2014 NAME:
EXAM 3

1. Consider the vector space $W \subseteq \mathbb{R}^3$ given by

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \,\middle|\, x - 2y + 3z = 0 \right\}.$$

Two ordered bases for W are

$$\mathcal{B} = \left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right).$$

(a) $\boxed{\text{10 pts.}}$ Let $\mathbf{v} = [5, 7, 3]^\top$, which is in W. Find $[\mathbf{v}]_\mathcal{B}$, the $\mathcal{B}$-coordinates of $\mathbf{v}$.

(b) $\boxed{\text{20 pts.}}$ Find a transition matrix $\mathbf{M}$ from coordinates in the basis $\mathcal{B}$ to coordinates in the basis $\mathcal{C}$, so that $\mathbf{M}[\mathbf{x}]_\mathcal{B} = [\mathbf{x}]_\mathcal{C}$ for all $\mathbf{x} \in W$.

(c) $\boxed{\text{5 pts.}}$ For the vector $\mathbf{v}$ above, use $\mathbf{M}$ to find $[\mathbf{v}]_\mathcal{C}$.

2. $\boxed{\text{15 pts.}}$ Suppose that $L : \mathbb{R}^2 \to \mathbb{R}^3$ is the linear transformation given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}.$$

Find $[L]_{\mathcal{BC}}$, the matrix corresponding to L with respect to the ordered bases

$$\mathcal{B} = \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right)$$

3. The matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

is diagonalizable.

(a) $\boxed{\text{10 pts.}}$ Find the characteristic equation of $\mathbf{A}$, and use it to find the eigenvalues of $\mathbf{A}$.

(b) $\boxed{\text{20 pts.}}$ For each eigenvalue of $\mathbf{A}$ find the bases for the corresponding eigenspaces.

(c) $\boxed{\text{10 pts.}}$ Find an invertible matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{P}^{-1}\mathbf{AP} = \mathbf{D}$.

4. Let W be the subspace of $\mathbb{R}^4$ spanned by the vectors

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}.$$

(a) $\boxed{\text{20 pts.}}$ Beginning with the vector $\mathbf{v}_1$, use the Gram-Schmidt Orthogonalization Process to obtain an orthogonal basis for W.

(b) $\boxed{\text{10 pts.}}$ Find an orthonormal basis for W.