MATH 260
SUMMER 2014
EXAM 2

NAME:

1. $\boxed{\text{10 pts. each}}$ Consider the system of equations

$$\begin{cases} 2x + y + z = 3 \\ x - y + 2z = 3 \\ x - 2y + \lambda z = 4 \end{cases}$$

Determine for which values of λ, if any, the system has:

(a) No solution.

(b) A unique solution, in which case give the solution.

(c) Infinitely many solutions, in which case give the solution.

2. $\boxed{\text{10 pts. each}}$ Let $\mathrm{Mat}_2(\mathbb{R})$ denote the vector space consisting of 2×2 matrices under the usual operations of matrix addition and scalar multiplication. Let

$$W_1 = \{\mathbf{A} \in \mathrm{Mat}_2(\mathbb{R}) : \mathbf{A}^\top = \mathbf{A}\} \quad \text{and} \quad W_2 = \left\{ \begin{bmatrix} a^2 & 0 \\ 0 & b^2 \end{bmatrix} : a, b \in \mathbb{R} \right\}.$$

(a) Prove or disprove that W_1 is a subspace of $\mathrm{Mat}_2(\mathbb{R})$.

(b) Prove or disprove that W_2 is a subspace of $\mathrm{Mat}_2(\mathbb{R})$.

3. $\boxed{\text{10 pts.}}$ Let

$$\mathbf{u}_1 = \begin{bmatrix} -1 \\ 3 \end{bmatrix} \quad \text{and} \quad \mathbf{u}_2 = \begin{bmatrix} 2 \\ -6 \end{bmatrix}.$$

Prove or disprove that $\mathrm{Span}\{\mathbf{u}_1, \mathbf{u}_2\} = \mathbb{R}^2$.

4. $\boxed{\text{10 pts. each}}$ Let

$$\mathbf{u}_1 = \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{u}_2 = \begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{u}_3 = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}.$$

(a) Show that $\{\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3\}$ is a linearly independent set.

(b) The ordered set $\mathcal{B} = (\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$ is a basis for $\mathbb{R}^3$. Given

$$\mathbf{v} = \begin{bmatrix} -6 \\ -10 \\ -5 \end{bmatrix},$$

find $[\mathbf{v}]_{\mathcal{B}}$, the coordinates of $\mathbf{v}$ with respect to the basis $\mathcal{B}$.

5. $\boxed{\text{10 pts. each}}$

(a) Write down a basis for the yz-plane in $\mathbb{R}^3$.

(b) The plane $x + 2y - 3z = 0$ is a subspace of $\mathbb{R}^3$. Find a basis for it.