MATH 260
SUMMER 2014
EXAM 1

NAME:

1. $\boxed{\text{10 pts. each}}$ Given that

$$\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 0 & -1 \\ -2 & 1 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -4 & 2 \\ 1 & -1 \\ 0 & 3 \end{bmatrix}$$

compute the following.

(a) $\mathbf{x}^\top \mathbf{x}$

(b) $\mathbf{x}\mathbf{x}^\top$

(c) $\mathbf{AC}$

2. $\boxed{\text{10 pts.}}$ Solve for the matrix $\mathbf{A}$:

$$\left(5\mathbf{A}^\top - \begin{bmatrix} 1 & 0 \\ 2 & 5 \end{bmatrix}\right)^\top = 3\mathbf{A} + \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}^{-1}$$

3. $\boxed{\text{10 pts.}}$ Find a vector equation for the line through the points $(4, 5, 1)$ and $(1, 3, -2)$.

4. $\boxed{\text{10 pts. each}}$ Let L_1 be the line given by

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad t \in \mathbb{R},$$

and let L_2 be the line with Cartesian equations

$$x = 5, \quad y - 4 = \frac{z-1}{2}.$$

(a) Show that the lines L_1 and L_2 intersect, and find the point of intersection.

(b) Find the equation of the plane containing L_1 and L_2.

5. $\boxed{\text{10 pts. each}}$ Let P be the plane in $\mathbb{R}^3$ which has normal vector $\mathbf{n} = [1, -4, 2]^\top$ and contains the point $(5, 1, 3)$.

(a) Find an algebraic equation for P.

(b) Find a vector equation for P.

6. $\boxed{\text{10 pts.}}$ Solve the system using Gaussian elimination to obtain a row-echelon form. Write the general solution in vector form.

$$\begin{cases} -3x - 5y + 36z = 10 \\ -x \qquad\quad + 7z = 5 \\ x + y - 10z = -4 \end{cases}$$

7. $\boxed{\text{15 pts.}}$ Find the inverse for the matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

using elementary row operations, then use the inverse to solve the system of equations given by $\mathbf{Ax} = \mathbf{b}$ for $\mathbf{b} = [2, 0, -1]^\top$.

8. $\boxed{\text{10 pts.}}$ Evaluate the determinant, using row or column operations to simplify the calculation:

$$\begin{vmatrix} 1 & -4 & 3 & 2 \\ 2 & -7 & 5 & 1 \\ 1 & 2 & 6 & 0 \\ 2 & -10 & 14 & 4 \end{vmatrix}.$$

9. $\boxed{\text{10 pts.}}$ Find the solution to the system

$$\begin{cases} x + y + z = 8 \\ 2x + y - z = 3 \\ -x + 2y + z = 3 \end{cases}$$

using Cramer's Rule.

10. $\boxed{\text{10 pts.}}$ Given that

$$\mathbf{A} = \begin{bmatrix} 2-\lambda & 3 \\ 2 & 1-\lambda \end{bmatrix},$$

for which real values of λ will the matrix equation $\mathbf{Ax} = \mathbf{0}$ have a nontrivial solution?