Math 260
Summer 2013 Name:
Exam 4

1. $\boxed{\text{10 pts.}}$ Compute the determinant

$$\begin{vmatrix} -1 & 1 & 2 & 0 \\ 0 & 3 & 2 & 1 \\ 0 & 4 & 1 & 2 \\ 3 & 1 & 5 & 7 \end{vmatrix}$$

by row or column expansions (i.e. use the definition of determinant).

2. $\boxed{\text{10 pts.}}$ Find the rank of the matrix

$$\begin{bmatrix} 3 & 5 & 1 & 4 \\ 2 & -1 & 1 & 1 \\ 8 & 9 & 3 & 9 \end{bmatrix}$$

using determinants.

3. $\boxed{\text{10 pts.}}$ Solve the system

$$\begin{cases} 2x - y + z = 1 \\ x + 3y - 2z = 0 \\ 4x - 3y + z = 2 \end{cases}$$

using Cramer's Rule.

4. $\boxed{\text{10 pts.}}$ Let $a \neq 0$. Prove that the eigenvectors of the matrix

$$\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$$

generate a 1-dimensional vector space, and give a basis for the space.

5. $\boxed{\text{15 pts.}}$ Let

$$\mathbf{A} = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$$

Use the characteristic polynomial to find the eigenvalues of $\mathbf{A}$, then find a basis for each eigenspace.

6. Let

$$\mathbf{B} = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$$

(a) $\boxed{\text{5 pts.}}$ Find the characteristic polynomial for $\mathbf{B}$.

(b) $\boxed{\text{5 pts.}}$ Find the eigenvalues of $\mathbf{B}$.

(c) $\boxed{\text{10 pts.}}$ Find bases for all eigenspaces of $\mathbf{B}$.

7. $\boxed{\text{10 pts.}}$ Find the eigenvalues of

$$\mathbf{C} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix},$$

and then find the maximum value of the associated quadratic form on the unit circle.

8. $\boxed{\text{10 pts.}}$ Let

$$\mathbf{A} = \begin{bmatrix} \lambda_1 & 0 & \cdots & 0 \\ 0 & \lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_n \end{bmatrix}$$

be an $n \times n$ diagonal matrix such that $\lambda_k \geq 0$ for $1 \leq k \leq n$. Show that there exists an $n \times n$ diagonal matrix $\mathbf{B}$ such that $\mathbf{B}^2 = \mathbf{A}$.