MATH 260
SUMMER 2013 NAME:
EXAM 2

1. $\boxed{\text{10 pts.}}$ Show that the set

$$\left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \middle| \; x^2 + y^2 + z^2 = 1 \right\}$$

is not a vector space under the operations inherited from $\mathbb{R}^3$.

2. $\boxed{\text{10 pts.}}$ The symbol $f : \mathbb{R} \to \mathbb{R}$ denotes a real-valued function of a single real-valued variable with domain $\mathbb{R}$. Prove or disprove that the set

$$\{f : \mathbb{R} \to \mathbb{R} \mid f(10) = 0\}$$

is a vector space under the usual operations of function addition and scalar multiplication established in basic algebra.

3. $\boxed{\text{10 pts.}}$ Show that the set

$$\{\langle x, y\rangle : x - 2y = 0\}$$

is a vector subspace of $\mathbb{R}^2$.

4. $\boxed{\text{10 pts.}}$ Prove that if U and W are subspaces of a vector space V, then $U \cap W$ is a subspace of V.

5. $\boxed{\text{10 pts.}}$ Let U be a subspace of $\mathbb{R}^n$. Prove that

$$U^\perp = \{\mathbf{x} \in \mathbb{R}^n : \mathbf{x} \cdot \mathbf{u} = 0 \text{ for all } \mathbf{u} \in U\}.$$

is also a subspace of $\mathbb{R}^n$.

6. $\boxed{\text{10 pts.}}$ Let $\mathbf{a} \in \mathbb{R}^n$ be a nonzero vector and let $c \in \mathbb{R}$ be fixed. Show that the set of all vectors $\mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{a} \cdot \mathbf{x} \geq c$ is convex.

7. $\boxed{\text{10 pts.}}$ Show that the vectors $\langle 1, 2\rangle$ and $\langle 1, 5\rangle$ are linearly independent.

8. $\boxed{\text{10 pts.}}$ Prove or disprove that the set of functions

$$\{\cos 2x, \cos^2 x, \sin^2 x\}$$

is linearly independent on $(-\infty, \infty)$.

9. $\boxed{\text{10 pts.}}$ Find the rank of

$$\begin{bmatrix} 1 & 1 & 0 & 1 \\ 1 & 2 & 2 & 1 \\ 3 & 4 & 2 & 3 \end{bmatrix}$$

by putting the matrix in either row echelon or column echelon form.

10. $\boxed{\text{10 pts.}}$ Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation defined by $T(x, y) = (xy, y)$. Describe the image under T of the set of points lying on the line $x = 2$.

11. $\boxed{\text{10 pts.}}$ Prove or disprove that the transformation $T : \mathbb{R}^2 \to \mathbb{R}$ given by $T(x, y) = xy$ is linear.

12. $\boxed{\text{10 pts.}}$ Show that the image of a convex set under a linear transformation is convex.

13. $\boxed{\text{10 pts.}}$ Let $L : V \to W$ be a linear transformation. Show that if S is an arbitrary line in V, then $L(S)$—the image of S under L—is either a point or a line in W.

Vector Space Axioms

A1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for any $\mathbf{u}, \mathbf{v} \in V$

A2. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ for any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$

A3. There exists some $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$ for any $\mathbf{u} \in V$

A4. For each $\mathbf{u} \in V$ there exists some $-\mathbf{u} \in V$ such that $-\mathbf{u} + \mathbf{u} = \mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

A5. For any $a \in \mathbb{R}$ and $\mathbf{u}, \mathbf{v} \in V$, $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$

A6. For any $a, b \in \mathbb{R}$ and $\mathbf{u} \in V$, $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$

A7. For any $a, b \in \mathbb{R}$ and $\mathbf{u} \in V$, $a(b\mathbf{u}) = (ab)\mathbf{u}$

A8. For all $\mathbf{u} \in V$, $1\mathbf{u} = \mathbf{u}$