Math 260
Summer 2013
Exam 1

Name:

1. [5 pts.] Given points $p = (3, -1, 8)$ and $q = (-2, -9, 0)$ in $\mathbb{R}^3$, find $-3p$, $p - q$, and $2p - 3q$.

2. [10 pts.] For what values of c, if any, are the vectors $\langle 2, 1, c\rangle$ and $\langle -3, 4, -1\rangle$ perpendicular?

3. [10 pts.] Let $\mathbf{u} \in \mathbb{R}^n$ be a vector orthogonal (i.e. perpendicular) to every vector $\mathbf{x}$ in $\mathbb{R}^n$. Show that $\mathbf{u} = \mathbf{0}$, the zero vector.

4. [10 pts. each] Let $\mathbf{u} = \langle 2, -1, 5\rangle$ and $\mathbf{v} = \langle -1, 1, 1\rangle$.
   (a) Find $\|\mathbf{u}\|$ and $\|\mathbf{v}\|$.
   (b) Find $\operatorname{proj}_{\mathbf{v}} \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

5. [15 pts.] Find the measure of the angle, to the nearest hundredth of a degree, between the diagonal of a cube and one of its edges.

6. [15 pts.] Prove that

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2$$

for any $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$.

7. [10 pts.] Find $\mathbf{AB}$ for

$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 0 \\ -1 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} -7 & 5 \\ 0 & 1 \\ 3 & -9 \end{bmatrix}.$$

8. [10 pts.] Let $c \in \mathbb{R}$ and $\mathbf{A}$ be a matrix. Show that $(c\mathbf{A})^\top = c\mathbf{A}^\top$.

9. Let $\mathbf{I}$ denote the 2×2 identity matrix, and $\mathbf{O}$ the 2×2 zero matrix.
   (a) [5 pts.] Find a 2×2 matrix $\mathbf{A}$ such that $\mathbf{A}^2 = -\mathbf{I}$.
   (b) [10 pts.] Determine all 2×2 matrices $\mathbf{A}$ such that $\mathbf{A}^2 = \mathbf{O}$.

10. [15 pts.] Using elementary row operations, find a row-equivalent matrix for

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{bmatrix}$$

that is in row echelon form.

11. 15 pts. Find the inverse for the matrix using elementary row operations:

$$\begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

12. 15 pts. Solve the system using Gaussian elimination:

$$\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$$

13. 15 pts. Solve the system using Gaussian elimination:

$$\begin{cases} -3x - 5y + 36z = 10 \\ -x + 7z = 5 \\ x + y - 10z = -4 \end{cases}$$