MATH 260
SUMMER 2012 NAME:
EXAM 1

1. $\boxed{\text{10 pts.}}$ Given points $p = (3, -1, 8)$ and $q = (-2, -9, 0)$ in $\mathbb{R}^3$, find $3p$, $p + q$, and $p - 2q$.

2. $\boxed{\text{10 pts.}}$ Given points $p = (-2, 0)$, $q = (8, -5)$, $r = (-11, -6)$, $s = (17, 1)$, find the equivalent located vectors: $\overrightarrow{pq}$, $\overrightarrow{pr}$, $\overrightarrow{rp}$, $\overrightarrow{ps}$, $\overrightarrow{qs}$, $\overrightarrow{sq}$. (It may help to plot the points.)

3. $\boxed{\text{10 pts.}}$ Let $\mathbf{u} \in \mathbb{R}^n$ be a vector orthogonal (i.e. perpendicular) to every vector $\mathbf{x}$ in $\mathbb{R}^n$. Show that $\mathbf{u} = \mathbf{0}$, the zero vector.

4. $\boxed{\text{10 pts. each}}$ Let $\mathbf{u} = \langle 2, -1, 5 \rangle$ and $\mathbf{v} = \langle -1, 1, 1 \rangle$.
   (a) Find $|\mathbf{u}|$ and $|\mathbf{v}|$.
   (b) Find $\operatorname{proj}_{\mathbf{v}} \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.
   (c) Find the angle between $\mathbf{u}$ and $\mathbf{v}$ to the nearest tenth of a degree.

5. $\boxed{\text{10 pts. each}}$ Let
$$\mathbf{A} = \begin{bmatrix} 2 & -3 & 0 \\ -1 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} -7 & 5 \\ 0 & 1 \\ 3 & -9 \end{bmatrix}$$
   (a) Find $\mathbf{A}^T$
   (b) Find $\mathbf{AB}$

6. $\boxed{\text{10 pts.}}$ Given
$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix},$$
find $\mathbf{A}^2$, $\mathbf{A}^3$, and $\mathbf{A}^4$.

7. $\boxed{\text{10 pts.}}$ Find the inverse for the matrix
$$\mathbf{R}_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

8. $\boxed{\text{10 pts.}}$ Show that the only solution to the system
$$\begin{cases} 4x - 7y + 3z = 0 \\ x + y = 0 \\ y - 6z = 0 \end{cases}$$
is the trivial solution.

9. $\boxed{\text{15 pts.}}$ Using elementary row operations, find a row-equivalent matrix for

$$\begin{bmatrix} 0 & 1 & 3 & -2 \\ 2 & 1 & -4 & 3 \\ 2 & 3 & 2 & -1 \end{bmatrix}$$

that is in row echelon form.

10. $\boxed{\text{15 pts.}}$ Using elementary row operations, find the inverse for the matrix

$$\begin{bmatrix} 2 & 4 & 3 \\ -1 & 3 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

11. $\boxed{\text{15 pts.}}$ Let V be the set of 2×2 matrices of the form

$$\begin{bmatrix} a & 0 \\ 1 & b \end{bmatrix}$$

with the standard operations of matrix addition and scalar multiplication. Determine whether V is a vector space. If it is not, indicate which axioms and closure properties fail to hold.

12. $\boxed{\text{10 pts.}}$ Show that the set of all $(x, y) \in \mathbb{R}^2$ such that $x + 4y = 0$ forms a subspace of $\mathbb{R}^2$.

13. $\boxed{\text{10 pts.}}$ Show that if U and W are subspaces of V, then $U + W$ is also a subspace of V.

14. $\boxed{\text{10 pts. each}}$ Determine whether the given vectors span $\mathbb{R}^3$.

(a) $\mathbf{v}_1 = \langle 1, 1, 1 \rangle$, $\mathbf{v}_2 = \langle 2, 2, 0 \rangle$, $\mathbf{v}_3 = \langle 3, 0, 0 \rangle$.

(b) $\mathbf{v}_1 = \langle 2, -1, 3 \rangle$, $\mathbf{v}_2 = \langle 4, 1, 2 \rangle$, $\mathbf{v}_3 = \langle 8, -1, 8 \rangle$.

Vector Space Axioms

A1. $\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$ for any $\mathbf{u}, \mathbf{v} \in V$

A2. $\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$ for any $\mathbf{u}, \mathbf{v}, \mathbf{w} \in V$

A3. There exists some $\mathbf{0} \in V$ such that $\mathbf{u} + \mathbf{0} = \mathbf{0} + \mathbf{u} = \mathbf{u}$ for any $\mathbf{u} \in V$

A4. For each $\mathbf{u} \in V$ there exists some $-\mathbf{u} \in V$ such that $-\mathbf{u} + \mathbf{u} = \mathbf{u} + (-\mathbf{u}) = \mathbf{0}$

A5. For any $a \in \mathbb{R}$ and $\mathbf{u}, \mathbf{v} \in V$, $a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$

A6. For any $a, b \in \mathbb{R}$ and $\mathbf{u} \in V$, $(a + b)\mathbf{u} = a\mathbf{u} + b\mathbf{u}$

A7. For any $a, b \in \mathbb{R}$ and $\mathbf{u} \in V$, $a(b\mathbf{u}) = (ab)\mathbf{u}$

A8. For all $\mathbf{u} \in V$, $1\mathbf{u} = \mathbf{u}$