

1. 20 pts. each The matrix

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & -2 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

is diagonalizable.

- (a) Find the characteristic equation of \mathbf{A} , and use it to find the eigenvalues of \mathbf{A} .
 - (b) For each eigenvalue of \mathbf{A} find the corresponding eigenvectors.
 - (c) Find an invertible matrix \mathbf{P} and a diagonal matrix \mathbf{D} such that $\mathbf{P}^{-1}\mathbf{A}\mathbf{P} = \mathbf{D}$.
2. 40 pts. Beginning with the vector \mathbf{v}_1 , use the Gram-Schmidt Orthonormalization Process to obtain an orthonormal basis for the subspace of \mathbb{R}^4 spanned by the following vectors:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 3 \\ 0 \\ 2 \\ 0 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 2 \\ 1 \\ -1 \\ 3 \end{bmatrix}.$$