MATH 260
SPRING 2014
EXAM 4

NAME:

1. Consider the vector space $W \subseteq \mathbb{R}^3$ given by

$$W = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \;\middle|\; x - 2y + 3z = 0 \right\}.$$

Two ordered bases for W are

$$\mathcal{B} = \left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix} \right).$$

(a) $\boxed{\text{10 pts.}}$ Show that $\mathbf{v} = [5,7,3]^\top$ is in W, and find $[\mathbf{v}]_{\mathcal{B}}$, the $\mathcal{B}$-coordinates of $\mathbf{v}$.

(b) $\boxed{\text{15 pts.}}$ Find a transition matrix $\mathbf{M}$ from coordinates in the basis $\mathcal{B}$ to coordinates in the basis $\mathcal{C}$, so that $\mathbf{M}[\mathbf{x}]_{\mathcal{B}} = [\mathbf{x}]_{\mathcal{C}}$ for all $\mathbf{x} \in W$.

(c) $\boxed{\text{10 pts.}}$ Use $\mathbf{M}$ to find $[\mathbf{v}]_{\mathcal{C}}$, where $\mathbf{v} = [5,7,3]^\top$.

2. $\boxed{\text{15 pts.}}$ Suppose that $L : \mathbb{R}^2 \to \mathbb{R}^3$ is the linear transformation given by

$$L\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \right) = \begin{bmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{bmatrix}.$$

Find $[L]_{\mathcal{BC}}$, the matrix corresponding to L with respect to the ordered bases

$$\mathcal{B} = \left(\begin{bmatrix} 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix} \right)$$

3. $\boxed{\text{10 pts. each}}$ For the matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & -2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

do the following:

(a) Find the characteristic polynomial $P_{\mathbf{A}}(t)$.

(b) Find all real eigenvalues for $\mathbf{A}$.

(c) Find a basis for the eigenspace corresponding to the smallest eigenvalue.

(d) Find a basis for the eigenspace corresponding to the largest eigenvalue.