MATH 260
SPRING 2014
EXAM 3

NAME:

1. $\boxed{\text{10 pts.}}$ Either prove or disprove that

$$U = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} : xyz = 0 \right\}.$$

is a subspace of $\mathbb{R}^3$.

2. $\boxed{\text{10 pts. each}}$ Let W_1 and W_2 be subspaces of a vector space V.

   (a) Show that $W_1 \cap W_2$ is a subspace of V.

   (b) Show that $W_1 \cup W_2$ is not necessarily a subspace of V, but if $W_1 \subseteq W_2$ or $W_2 \subseteq W_1$ is given, then $W_1 \cup W_2$ is a subspace V.

3. $\boxed{\text{15 pts.}}$ Show that the set

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

spans $\mathbb{R}^3$, but any vector $\mathbf{v} \in \mathbb{R}^3$ can be written as a linear combination of vectors in S in infinitely many ways.

4. $\boxed{\text{15 pts.}}$ Show that the vectors

$$\mathbf{x}_1 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 3 \\ 4 \\ 6 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} -2 \\ 3 \\ 2 \end{bmatrix}$$

are linearly independent. Express the vector

$$\mathbf{v} = \begin{bmatrix} -2 \\ 0 \\ 6 \end{bmatrix}$$

as a linear combination of them.

5. $\boxed{\text{10 pts.}}$ The plane $x - 2y + z = 0$ is a subspace of $\mathbb{R}^3$. Find a basis for it.