MATH 260
SPRING 2014 NAME:
EXAM 1

1. [10 pts. each] Given that

$$\mathbf{x} = \begin{bmatrix} 3 \\ -1 \\ 2 \end{bmatrix}, \quad \mathbf{A} = \begin{bmatrix} 1 & 2 & -3 \\ 3 & 0 & -1 \\ -2 & 1 & 4 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} -4 & 2 \\ 1 & -1 \\ 0 & 3 \end{bmatrix}$$

compute the following.

(a) $\mathbf{x}^\top \mathbf{x}$

(b) $\mathbf{x}\mathbf{x}^\top$

(c) $\mathbf{AC}$

2. [10 pts.] Solve for the matrix $\mathbf{A}$:

$$\left(4\mathbf{A}^\top - \begin{bmatrix} 9 & 0 \\ -3 & 2 \end{bmatrix}\right)^\top = 3\mathbf{A} + \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}^{-1}$$

3. [10 pts. each] Write a vector equation for each of the following.

(a) In $\mathbb{R}^2$, the line through the points $(4, -2)$ and $(3, 7)$.

(b) In $\mathbb{R}^4$, the line through the points $(3, 0, -1, 2)$ and $(-2, 6, 0, 1)$. Is the point $(-7, 12, 1, 0)$ on this line?

4. [10 pts. each] Let L_1 be the line given by

$$\mathbf{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + t \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad t \in \mathbb{R},$$

and let L_2 be the line with Cartesian equations

$$x = 5, \quad y - 4 = \frac{z - 1}{2}.$$

(a) Show that the lines L_1 and L_2 intersect, and find the point of intersection.

(b) Find the equation of the plane containing L_1 and L_2.

5. [10 pts.] Solve the system using Gaussian elimination, obtaining reduced row-echelon form as in the textbook.

$$\begin{cases} x + 2y - z = 9 \\ 2x - z = -2 \\ 3x + 5y + 2z = 22 \end{cases}$$

6. [10 pts.] Solve the system using Gaussian elimination, obtaining either row-echelon or reduced row-echelon form. Write the general solution in vector form.

$$\begin{cases} -3x - 5y + 36z = 10 \\ -x \quad\quad + 7z = 5 \\ x + y - 10z = -4 \end{cases}$$

7. [10 pts. each] Let P_1 and P_2 be the planes

$$3x + y + z = 4 \quad \text{and} \quad x - 2y - z = 1,$$

respectively.

(a) Find the vector equation of the line of intersection of P_1 and P_2.

(b) What is the intersection of the planes P_1 and P_2 and the plane $x + 2y + 2z = 1$?