MATH 260
FALL 2018
EXAM 3

NAME:

1. [10 pts.] Let $L : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear mapping such that $L(1,1) = (2,-1)$ and $L(-1,3) = (1,2)$. Find $L(0,1)$.

2. [10 pts.] Let V, W be vector spaces and $L : V \to W$ a linear mapping. Suppose $\mathbf{w}_1, \ldots, \mathbf{w}_n \in W$ are linearly independent and $\mathbf{v}_1, \ldots, \mathbf{v}_n \in V$ are such that $L(\mathbf{v}_k) = \mathbf{w}_k$ for $1 \leq k \leq n$. Show that $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent.

3. [10 pts.] Let $L : V \to W$ be a linear mapping. Assume $\dim V > \dim W$. Prove that the kernel of L is not $\{\mathbf{0}\}$.

4. [10 pts.] Find the dimension of the subspace of $\mathbb{R}^5$ orthogonal to the vectors $(1,1,-2,3,4)$, $(1,0,0,2,0)$, $(0,1,0,1,0)$.

5. [10 pts.] Find the matrix corresponding to the linear mapping $L : \mathbb{R}^4 \to \mathbb{R}^3$ given by

$$L\big([x_1, x_2, x_3, x_4]^\top\big) = [2x_3, 0, -2x_1]^\top$$

with respect to the standard bases.

6. [10 pts.] Let $P : V \to V$ be a linear map such that $P \circ P = P$. Show that $V = \mathrm{Ker}(P) + \mathrm{Img}(P)$.

7. [10 pts.] Let $L : \mathbb{R}^{1\times 2} \to \mathbb{R}^{1\times 2}$ be the linear transformation defined by

$$L(x, y) = [2x + y, 3x - 5y].$$

Show that L is invertible.

8. [10 pts.] The ordered sets

$$\mathcal{B} = \left(\begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right) \quad \text{and} \quad \mathcal{C} = \left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right)$$

are bases for $\mathbb{R}^2$. Find the change of basis matrix $\mathbf{I}_{\mathcal{BC}}$ (a.k.a. transition matrix) for changing from the basis $\mathcal{B}$ to the basis $\mathcal{C}$.