Math 260
Fall 2018
Exam 2

Name:

1. 10 pts. Prove or disprove that the set of all $[x, y, z]$ such that $x - 2y = 4z$ is a subspace of $\mathbb{R}^3$.

2. 10 pts. Let V be a subspace of $\mathbb{R}^n$, and let W be the set of vectors in $\mathbb{R}^n$ which are perpendicular to every vector in V. Show that W is a subspace of $\mathbb{R}^n$.

3. 10 pts. Prove or disprove that the vectors $[1, 2, 0]$, $[1, 3, -1]$, $[-1, 1, 1]$ are linearly independent vectors in $\mathbb{R}^3$.

4. 10 pts. Let V be a vector space with $\mathbf{v}_1, \mathbf{v}_2 \in V$, and let S be the parallelogram consisting of all linear combinations $t_1\mathbf{v}_1 + t_2\mathbf{v}_2$ with $0 \le t_1 \le 1$ and $0 \le t_2 \le 1$. Prove that S is convex.

5. 10 pts. Find the coordinates of the vector $\mathbf{x} = [1, 1, 1]$ with respect to the vectors $\mathbf{u}_1 = [0, 1, -1]$, $\mathbf{u}_2 = [1, 1, 0]$, and $\mathbf{u}_3 = [1, 0, 2]$.

6. 10 pts. Let $\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_m$ be nonzero vectors in $\mathbb{R}^n$, and assume they are mutually perpendicular (i.e. $\mathbf{v}_i \perp \mathbf{v}_j$ whenever $i \neq j$). Prove the vectors are linearly independent.

7. 10 pts. The plane P given by $2x - y + 3z = 0$ is a subspace of $\mathbb{R}^3$. Find a basis for P.

8. 10 pts. What is the dimension of the vector space consisting of symmetric $n \times n$ matrices?

9. 10 pts. Find the rank of

$$\begin{bmatrix} 1 & 2 & -3 \\ -1 & -2 & 3 \\ 4 & 8 & -12 \\ 1 & -1 & 5 \end{bmatrix}$$

by obtaining either a row echelon or column echelon form.

10. 10 pts. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the transformation defined by $T(x, y) = [3x, 7y]$. Describe the image under T of the points lying on the circle $x^2 + y^2 = 1$.