MATH 260
FALL 2017
EXAM 4

NAME:

1. $\boxed{\text{10 pts.}}$ An $n \times n$ symmetric matrix $\mathbf{A}$ with real entries is called positive definite if $\mathbf{x}^\top \mathbf{A}\mathbf{x} > 0$ for all $\mathbf{x} \in \mathbb{R}^n$ such that $\mathbf{x} \neq \mathbf{0}$. Prove or disprove that

$$\mathbf{A} = \begin{bmatrix} 4 & 1 \\ 1 & 10 \end{bmatrix}$$

is positive definite.

2. $\boxed{\text{10 pts.}}$ Find an orthonormal basis for the subspace of $\mathbb{R}^3$ generated by the vectors $(2, 1, 1)$ and $(1, 3, -1)$.

3. $\boxed{\text{15 pts.}}$ Let V be the vector space of all continuous functions $f : [0, 1] \to \mathbb{R}$, and define a scalar product on V by

$$\langle f, g \rangle = \int_0^1 f(t)g(t)\,dt$$

for all $f, g \in V$. Let U be the subspace of V generated by $\{1, t, t^2, t^3\}$; that is, $U = \operatorname{Span}\{1, t, t^2, t^3\}$. Find an orthogonal basis for U.

4. $\boxed{\text{10 pts.}}$ Evaluate the determinant:

$$\begin{vmatrix} 4 & -2 & 3b \\ 0 & a & -13 \\ 1 & 0 & a \end{vmatrix}.$$

5. $\boxed{\text{10 pts.}}$ Evaluate the determinant, which will be a polynomial in t:

$$\begin{vmatrix} -2 & 2 & 3 & -4 \\ 1 & 1 & 6 & 3 \\ -1 & 0 & 1 & -1 \\ -2 & t & 6 & 5 \end{vmatrix}.$$

6. $\boxed{\text{15 pts.}}$ Solve the system using Cramer's Rule.

$$\begin{cases} x + 2y - 3z + 5w = 0 \\ 2x + y - 4z - w = 1 \\ x + y + z + w = 0 \\ -x - y - z + w = 4 \end{cases}$$

7. $\boxed{\text{10 pts.}}$ Use determinants to find the rank of the matrix

$$\mathbf{H} = \begin{bmatrix} 3 & 5 & 1 & 4 \\ 2 & -1 & 1 & 1 \\ 8 & 9 & 3 & 9 \end{bmatrix}.$$