MATH 260
FALL 2017
EXAM 2

NAME:

Vector Space Axioms:
VS1. $\mathbf{u}+\mathbf{v}=\mathbf{v}+\mathbf{u}$ for any $\mathbf{u},\mathbf{v}\in V$
VS2. $\mathbf{u}+(\mathbf{v}+\mathbf{w})=(\mathbf{u}+\mathbf{v})+\mathbf{w}$ for any $\mathbf{u},\mathbf{v},\mathbf{w}\in V$
VS3. There exists some $\mathbf{0}\in V$ such that $\mathbf{u}+\mathbf{0}=\mathbf{u}$ for any $\mathbf{u}\in V$
VS4. For each $\mathbf{u}\in V$ there exists some $-\mathbf{u}\in V$ such that $\mathbf{u}+(-\mathbf{u})=\mathbf{0}$
VS5. For any $a\in\mathbb{F}$ and $\mathbf{u},\mathbf{v}\in V$, $a(\mathbf{u}+\mathbf{v})=a\mathbf{u}+a\mathbf{v}$
VS6. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $(a+b)\mathbf{u}=a\mathbf{u}+b\mathbf{u}$
VS7. For any $a,b\in\mathbb{F}$ and $\mathbf{u}\in V$, $a(b\mathbf{u})=(ab)\mathbf{u}$
VS8. For all $\mathbf{u}\in V$, $1\mathbf{u}=\mathbf{u}$

1. $\boxed{\text{10 pts.}}$ Show that the set of vectors $[x,y,z]$ in $\mathbb{R}^3$ that satisfy $x-2y+3z=0$ is a subspace of $\mathbb{R}^3$.

2. $\boxed{\text{10 pts.}}$ Prove or disprove that the set of vectors $[x,y]$ in $\mathbb{R}^2$ that satisfy $x-2y^2=0$ is a subspace of $\mathbb{R}^2$.

3. $\boxed{\text{10 pts.}}$ Let V be a subspace of $\mathbb{R}^n$, and let W be the set of vectors in $\mathbb{R}^n$ which are perpendicular (i.e. orthogonal) to every vector in V. Show that W is a subspace of $\mathbb{R}^n$.

4. $\boxed{\text{10 pts.}}$ Prove or disprove that the vectors $[1,2,0]$, $[1,3,-1]$, $[-1,1,1]$ are linearly independent vectors in $\mathbb{R}^3$.

5. $\boxed{\text{10 pts.}}$ Let V be a vector space with $\mathbf{v}_1,\mathbf{v}_2\in V$, and let S be the parallelogram consisting of all linear combinations $t_1\mathbf{v}_1+t_2\mathbf{v}_2$ with $0\le t_1\le 1$ and $0\le t_2\le 1$. Prove that S is convex.

6. $\boxed{\text{10 pts.}}$ The vectors $\mathbf{u}_1=[2,1]$ and $\mathbf{u}_2=[-1,0]$ form a basis for $\mathbb{R}^2$. Find the coordinates of $\mathbf{x}=[4,-3]$ with respect to $\mathbf{u}_1$, $\mathbf{u}_2$.

7. $\boxed{\text{10 pts.}}$ In #1 it was shown that $V=\{[x,y,z]:x-2y+3z=0\}$ is a subspace of $\mathbb{R}^3$. Find a basis for V.

8. $\boxed{\text{10 pts.}}$ Consider the vector space of functions defined on the interval $(0,\infty)$. Prove or disprove that the set of functions $\{e^t,\ln t\}$ is linearly independent.

9. $\boxed{\text{10 pts.}}$ Find the rank of the matrix, justifying your answer:

$$\mathbf{M}=\begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \\ 3 & 8 & 2 & -12 \end{bmatrix}.$$

10. $\boxed{\text{15 pts.}}$ Let $\mathbf{A}$ be an $m \times n$ matrix and $\mathbf{B}$ an $n \times r$ matrix. Show that the columns of $\mathbf{AB}$ are a linear combination of the columns of $\mathbf{A}$, and go on to prove that $\operatorname{rank}(\mathbf{AB}) \leq \operatorname{rank}(\mathbf{A})$.

11. $\boxed{\text{15 pts.}}$ Let $F : \mathbb{R}^2 \to \mathbb{R}^2$ be the mapping defined by

$$F(x, y) = [e^{-x} \cos y, e^{-x} \sin y].$$

Describe the image under F of the line $x = c$, where c is a constant. Also describe the image under F of the line $y = d$, where d is a constant.