MATH 260
FALL 2017
EXAM 1

NAME:

1. Let $\mathbf{u} = [3, -1, 4]$.
   (a) [5 pts.] Find a vector parallel to $\mathbf{u}$ that has length 1.
   (b) [10 pts.] Find a vector orthogonal to $\mathbf{u}$ that has length 4.

2. Let $\mathbf{u} = [-4, 0, 2]$ and $\mathbf{v} = [-2, -1, 1]$.
   (a) [10 pts.] Find $\mathrm{proj}_{\mathbf{v}}\, \mathbf{u}$, the orthogonal projection of $\mathbf{u}$ onto $\mathbf{v}$.
   (b) [5 pts.] Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

3. [10 pts.] For vectors $\mathbf{u}, \mathbf{v} \in \mathbb{R}^n$ use basic properties of the dot product to prove that

$$\|\mathbf{u} + \mathbf{v}\|^2 + \|\mathbf{u} - \mathbf{v}\|^2 = 2\|\mathbf{u}\|^2 + 2\|\mathbf{v}\|^2.$$

4. [10 pts.] Find a parametric equation for the line passing through the points $p = (1, 0, -1)$ and $q = (2, 2, -3)$.

5. [10 pts.] Find an equation of the plane in $\mathbb{R}^3$ that is perpendicular to $\mathbf{n} = [-3, 2, 8]$ and contains the point $p = (1/2, 0, 2)$.

6. [10 pts.] Find a parametric equation $\mathbf{x}(t)$ for the line of intersection of the two planes $x - y + z = 2$ and $2x - 3y + z = 6$.

7. [10 pts.] Find $\mathbf{ABC}$ for

$$\mathbf{A} = \begin{bmatrix} 2 & a & 1 \\ 3 & -5 & 2 \end{bmatrix}, \quad \mathbf{B} = \begin{bmatrix} 1 & 3 \\ 5 & -3 \\ a & -1 \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

8. [10 pts.] Suppose that $\mathbf{A}^3 - \mathbf{A} + \mathbf{I} = \mathbf{O}$. Show that $\mathbf{A}$ is invertible.

9. Let $\mathbf{A}$ and $\mathbf{B}$ be two $n \times n$ matrices. We say $\mathbf{A}$ is **similar** to $\mathbf{B}$ if there exists an invertible matrix $\mathbf{T}$ such that $\mathbf{B} = \mathbf{T}\mathbf{A}\mathbf{T}^{-1}$. Suppose this is the case. Prove the following.
   (a) [5 pts.] $\mathbf{B}$ is similar to $\mathbf{A}$.
   (b) [10 pts.] $\mathbf{A}$ is invertible if and only if $\mathbf{B}$ is invertible.

10. [10 pts.] Find the inverse for the matrix using elementary row operations, if the inverse exists:

$$\mathbf{C} = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$$

11. [10 pts.] Find the solution set of the system using Gaussian elimination:

$$\begin{cases} x + 2y + 2z = 9 \\ 2x + 4y - 3z = 1 \\ 3x + 6y - 5z = 0 \end{cases}$$

12. [10 pts.] Find the complete solution set of the system:

$$\begin{cases} 3x - y + 6z = 4 \\ x + y - 2z = 0 \end{cases}$$