1 Solve the general solution to $y'' + 3y = -48x^2 e^{3x}$.

Auxiliary equation is $r^2 + 3 = 0$, with solution $r = \pm i\sqrt{3}$. Particular solution will have form $y_p = (Ax^2 + Bx + C)e^{3x}$. We then find that

$$y_p'' + 3y_p = (12Ax^2 + 12Ax + 12Bx + 2A + 6B + 12C)e^{3x},$$

and so A, B, C must be such that

$$12Ax^2 + 12Ax + 12Bx + 2A + 6B + 12C = -48x^2.$$

This gives us the system

$$\begin{cases} 12A & = -48 \\ 12A + 12B & = 0 \\ 2A + 6B + 12C = & 0 \end{cases}$$

which has solution $A = -4$, $B = 4$, $C = -\frac{4}{3}$. General solution is therefore

$$y = \left(-4x^2 + 4x - \frac{4}{3}\right)e^{3x} + c_1 \cos\sqrt{3}x + c_2 \sin\sqrt{3}x.$$