Math 250 Quiz #2 (Fall 2020)

1 Solve the Bernoulli equation $\dfrac{dy}{dx} - y = e^x y^2$.

Let $v = y^{-1}$, so $y = 1/v$, and $v' = -y^{-2}y'$ implies $y' = -v'/v^2$. Putting these results into the equation yields the linear equation $v' + v = -e^x$. Integrating factor is $\mu(x) = e^x$, and then

$$v' + v = -e^x \quad \Rightarrow \quad e^x v' + e^x v = -e^{2x} \quad \Rightarrow \quad (e^x v)' = -e^{2x} \quad \Rightarrow \quad e^x v = -\frac{1}{2}e^{2x} + c.$$

From this we get

$$v = -\frac{1}{2}e^x + ce^{-x},$$

and finally

$$y = \frac{1}{ce^{-x} - e^x/2} = \frac{2}{ce^{-x} - e^x}.$$

2 Solve $\dfrac{dy}{dx} = \cos(x+y)$.

Let $u = x + y$, so $y' = u' - 1$. The differential equation becomes $u' = 1 + \cos u$, which is separable and gives

$$\int \frac{1}{1+\cos u}\,du = \int dx \quad \Rightarrow \quad \int \frac{1}{1+\cos u} \cdot \frac{1-\cos u}{1-\cos u}\,du = x + c.$$

Thus we have

$$x + c = \int \frac{1-\cos u}{\sin^2 u}\,du = \int (\csc^2 u - \cot u \csc u)\,du = -\cot u + \csc u,$$

and therefore

$$\csc(x+y) - \cot(x+y) = x + c.$$