MATH 250 QUIZ #1 (FALL 2020)

1a Verify that $y = (1 - \sin x)^{-1/2}$ is an explicit solution to $2y' = y^3 \cos x$.

For y as given we have

$$2y' = 2\left[-\frac{1}{2}(1 - \sin x)^{-3/2}(-\cos x)\right] = \frac{\cos x}{(1 - \sin x)^{3/2}},$$

while

$$y^3 \cos x = \left[(1 - \sin x)^{-1/2}\right]^3 \cos x = \frac{\cos x}{(1 - \sin x)^{3/2}}.$$

The two sides of the differential equation thus match, verifying $y = (1 - \sin x)^{-1/2}$ is a solution.

1b Give an interval I of definition for the solution given in part (a).

The solution $y = (1 - \sin x)^{-1/2}$ has domain $\{x : x \neq \frac{\pi}{2} + 2\pi n\}$, where n is any integer. Two choices for I are $(\frac{\pi}{2}, \frac{5\pi}{2})$ and $(-\frac{3\pi}{2}, \frac{\pi}{2})$.

2 Determine the region in the xy-plane where the IVP $(y - x^2)y' = y + \sqrt{x}$, $y(x_0) = y_0$, must have a unique solution.

Rewrite the differential equation as $y' = f(x, y)$, with

$$f(x, y) = \frac{y + \sqrt{x}}{y - x^2}.$$

The function f is continuous on its domain, which is $\{x : x \geq 0 \text{ and } y \neq x^2\}$. The function

$$\frac{\partial f}{\partial y}(x, y) = -\frac{x^2 + \sqrt{x}}{(y - x^2)^2}$$

is continuous on the same domain. By the Existence-Uniqueness Theorem we conclude that the IVP has a unique solution for

$$(x_0, y_0) \in \{x : x > 0 \text{ and } y \neq x^2\}.$$