

MATH 250
SUMMER 2024
EXAM 3

NAME:

1. 20 pts. Find the general solution to $y''' - 3y'' + 4y' - 2y = 0$.

2. 20 pts. Find the solution to the initial-value problem

$$y''' - y' = 1, \quad y(0) = y'(0) = y''(0) = 4.$$

3. 20 pts. A force of 400 newtons stretches a spring 2 meters. A mass of 50 kg is attached to the end of the spring and is initially released from the equilibrium position with an upward velocity of 10 m/s. Find the equation of motion, and put it in the form $x(t) = A \sin(\omega t + \varphi)$. What is the period of motion? At what times is the mass heading downward at a velocity of 5 m/s?

4. 10 pts. Given that

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n$$

for $|x| < 1$, use differentiation to find a power series for $1/(1-x)^2$.

5. 20 pts. Find a power series solution of the form $\sum c_k x^k$ to $y' - y = x^2$.

6. 20 pts. Find two linearly independent power series solutions to

$$y'' + xy' + 2y = 0$$

about the ordinary point $x = 0$. Determine at least the first four nonzero terms of each series.

Method of Undetermined Coefficients. Let $P_m(x)$ be a nonzero polynomial of degree m , and let $y_p(x)$ denote a particular solution to $a_n y^{(n)} + \cdots + a_1 y' + a_0 y = f(x)$.

1. If $f(x) = P_m(x)e^{\alpha x}$, then

$$y_p(x) = x^s e^{\alpha x} \sum_{k=0}^m A_k x^k,$$

where $s = 0$ if α is not a root of the auxiliary equation, otherwise s equals the multiplicity of α as a root of the auxiliary equation.

2. If $f(x) = P_m(x)e^{\alpha x} \cos \beta x$ or $f(x) = P_m(x)e^{\alpha x} \sin \beta x$ for $\beta \neq 0$, then

$$y_p(x) = x^s e^{\alpha x} \left(\cos \beta x \sum_{k=0}^m A_k x^k + \sin \beta x \sum_{k=0}^m B_k x^k \right),$$

where $s = 0$ if $\alpha + i\beta$ is not a root of the auxiliary equation, otherwise s equals the multiplicity of $\alpha + i\beta$ as a root of the auxiliary equation.