MATH 250
SUMMER 2024 NAME:
EXAM 1

1. [10 pts.] Find all values of m for which $y = x^m$ is a solution to $x^2y'' - 7xy' + 15y = 0$.

2. [10 pts.] Solve the differential equation $y' + 2xy^2 = 0$.

3. [10 pts.] Find an explicit solution to the initial-value problem

$$x^2\frac{dy}{dx} = y - xy, \quad y(-1) = -1.$$

4. [10 pts.] Solve the linear differential equation

$$x^2y' + x(x+2)y = e^x.$$

5. [10 pts.] A Bernoulli equation has the form $y' + p(x)y = q(x)y^n$, which is linear if $n = 0$ or $n = 1$. If $n \neq 0, 1$, the equation can be transformed into a linear equation by making the substitutiion $u = y^{1-n}$. Solve the Bernoulli equation $y' - y = e^xy^2$.

6. [10 pts.] Solve the exact differential equation

$$(2xy^4 + \sin y)dx + (4x^2y^3 + x\cos y)dy = 0.$$

7. [10 pts. each] Consider the family $\mathcal{F}$ of curves $x^2 - 2y^2 = c$.
   (a) Find the differential equation of $\mathcal{F}$.
   (b) Find the orthogonal trajectories of $\mathcal{F}$.

8. [10 pts.] The differential equation in the initial-value problem

$$xy^2\frac{dy}{dx} = y^3 - x^3, \quad y(1) = 2$$

is homogeneous. Solve the IVP.

Some Integration Formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}}\,dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x\,dx = \ln|\sec x| + c, \quad \int \cot x\,dx = \ln|\sin x| + c, \quad \int \sec x\,dx = \ln|\sec x + \tan x| + c,$$

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + c.$$