MATH 250
SUMMER 2022 NAME:
EXAM 4

1. 20 pts. Solve the system using operator notation and elimination:

$$\begin{cases} 2\dfrac{dx}{dt} - 5x + \dfrac{dy}{dt} = e^t \\ \dfrac{dx}{dt} - x + \dfrac{dy}{dt} = 5e^t \end{cases}$$

2. 20 pts. Solve the system by the method of Laplace transforms:

$$\begin{cases} \dfrac{dx}{dt} - 2y = e^t \\ 8x - \dfrac{dy}{dt} = t, \end{cases}$$

where $x(0) = 1$ and $y(0) = 1$.

3. 20 pts. There are three tanks, A, B, and C, each with 100 L of a potassium chloride (KCl) solution. At time $t = 0$ there are x_0, y_0, and z_0 kg of KCl in A, B, and C, respectively. Tank A is linked to B by two pipes P_1 and P_2, and B is linked to C by two pipes P_3 and P_4. Also, A has an inlet pipe and C has a drain. The inlet pipe is pumping 4 L/min of pure water into A, and the drain is removing 4 L/min of KCl solution from C. Meanwhile, P_1 is moving 6 L/min from A to B, and P_2 is moving 2 L/min from B to A. Finally, P_3 is moving 5 L/min from B to C, and P_4 is moving 1 L/min from C to B. Letting $x(t)$, $y(t)$, and $z(t)$ be the mass of KCl at time t in A, B, and C, set up a system of differential equations that models the number of kilograms of KCl in each tank over time. *Do not solve the system!*

4. 20 pts. Two walls, W_1 and W_2, face each other across a smooth floor. A spring with stiffness k_1 is attached to W_1 on one end and a mass m_1 (on the floor) at the other end. A second spring with stiffness k_2 is attached to mass m_1 on one end and a mass m_2 (also on the floor) at the other end. Finally, a third spring with stiffness k_3 is attached to m_2 on one end and W_2 at the other end. Thus there is a linear arrangement of two masses and three springs on the floor, anchored on either end by a wall. Assuming $x_1 = 0$ and $x_2 = 0$ are the equilibrium positions for mass m_1 and m_2, respectively, derive a system of differential equations involving functions $x_1(t)$ and $x_2(t)$ which give the positions of m_1 and m_2 over time t. *Do not solve the system!*

5. 20 pts. Obtain terms to order 4, in powers of x, of the particular solution to the IVP

$$(x + 2)y'' + 3y = 0, \quad y(0) = 0, \quad y'(0) = 1.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\dots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$

$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$

$\mathcal{L}[e^{at}f(t)](s)=\mathcal{L}[f(t)](s-a)$

$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

$\cos^2 x=\frac{1}{2}(1+\cos 2x)$

$\sin x\cos y=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$