Math 250
Summer 2022 Name:
Exam 3

1. [20 pts.] Find the general solution to $y' - 2y = e^{2x}$ by the method of Laplace transforms by letting $y(0) = c$ for arbitrary constant c.

2. [20 pts.] Solve the initial-value problem by the method of Laplace transforms:

$$y'' + 4y' - 5y = xe^x, \quad y(0) = 1, \quad y'(0) = 0.$$

3. A particle exhibits simple harmonic motion. Every 0.4 second it passes through the equilibrium position with a velocity of ± 6 m/s.

   (a) [15 pts.] Set up and solve a differential equation to find the particle's equation of motion. Put the equation in the form $x(t) = A\sin(\omega t + \varphi)$.

   (b) [5 pts.] Find the period, natural frequency, and amplitude of the motion.

4. A 0.125 kg object is attached to a spring with stiffness $k = 16$ N/m. The object is displaced 0.5 m to the right of the equilibrium position (thereby stretching the spring) and given a rightward velocity of $\sqrt{2}$ m/s. There is no damping.

   (a) [15 pts.] Set up and solve a differential equation to find the object's equation of motion. Put the equation in the form $x(t) = A\sin(\omega t + \varphi)$.

   (b) [5 pts.] Find the period, natural frequency, and amplitude of the motion.

   (c) [5 pts.] When does the object first pass through the equilibrium position?

   (d) [5 pts.] What is the object's maximum displacement from the equilibrium position?

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$

$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$

$\mathcal{L}[e^{at}f(t)](s)=\mathcal{L}[f(t)](s-a)$

$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

$\cos^2 x=\frac{1}{2}(1+\cos 2x)$

$\sin x\cos y=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$