1. [20 pts.] By natural increase alone a population of mice on an island devoid of cats doubles every month, the increase proceeding at a rate proportional to the number of mice at any time t. At the beginning of 1990 the number of mice reached 100,000, at which point cats were introduced to the island. Thereafter the cats killed 1000 mice per month, offsetting the natural increase of the population of mice. How many mice are on the island 24 months later, at the beginning of 1992?

2. [15 pts.] Find the particular solution to the IVP

$$y'' - 2y' - 3y = 0, \quad y(0) = -1, \quad y'(0) = 2.$$

3. [10 pts.] Find the general solution to $y^{(4)} + 8y'' + 16y = 0$.

4. [10 pts.] Find the general solution to $y^{(5)} - 2y^{(4)} + y^{(3)} = 0$.

5. [10 pts.] Solve using the method of undetermined coefficients:

$$y'' + 2y' + 10y = 5xe^{-2x}.$$

6. [10 pts.] Solve using the method of undetermined coefficients:

$$y''' + y' = 2x^2 + 4\sin x.$$

7. [10 pts.] Solve using the method of variation of parameters:

$$y'' + 6y' + 9y = \frac{e^{-3x}}{x^3}.$$

Some Integration Formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}}\, dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}}\, dx = \frac{1}{a}\sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x\, dx = \ln|\sec x| + c, \quad \int \cot x\, dx = \ln|\sin x| + c, \quad \int \sec x\, dx = \ln|\sec x + \tan x| + c,$$

$$\int \csc x\, dx = -\ln|\csc x + \cot x| + c.$$