

MATH 250
SUMMER 2022
EXAM 1

NAME:

1. 10 pts. Find a differential equation having $x^2 + (y + c)^2 = 1 + c^2$ as a one-parameter family of solutions.

2. 10 pts. Solve the initial-value problem

$$\frac{dy}{dx} = \frac{\cos x}{3y^2 + e^y}, \quad y(0) = 2.$$

3. 10 pts. Solve the homogeneous equation

$$2x^2 \frac{dy}{dx} = x^2 + y^2.$$

Put the solution in explicit form and state the intervals of validity for the solution.

4. 10 pts. Solve the exact equation

$$2x + e^y + xe^y \frac{dy}{dx} = 0,$$

leaving the solution in an implicit form (i.e. don't isolate y).

5. 10 pts. Solve the linear initial-value problem

$$xy' - 3y = x^3, \quad y(1) = 0,$$

writing the solution in explicit form.

6. 10 pts. Solve the Bernoulli equation

$$y' + \frac{3}{x}y = x^2y^2.$$

7. 15 pts. A tank initially has 500 L of a solution containing 40 kg of isopropyl alcohol. Pure water flows into the tank at the rate of 3 L/min while the alcohol solution is drained out at the rate of 2 L/min. When will there be 25 kg of isopropyl alcohol in the tank?

8. 15 pts. An object with a temperature of 10°F is placed in a room where the temperature is 80°F . After 10 minutes the temperature of the object is 30°F . What will be the temperature of the object after it has been in the room for 30 minutes?

9. 10 pts. Prove that two functions are linearly dependent on an interval I if one function is a constant multiple of the other on I .

Some Integration Formulas:

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + c, \quad \int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \sec^{-1}\left|\frac{x}{a}\right| + c,$$

$$\int \tan x dx = \ln |\sec x| + c, \quad \int \cot x dx = \ln |\sin x| + c, \quad \int \sec x dx = \ln |\sec x + \tan x| + c,$$

$$\int \csc x dx = -\ln |\csc x + \cot x| + c.$$