

1. 10 pts. Find the values of m so that $y = x^m$ is a solution to

$$x^2 y'' - 7xy' + 15y = 0.$$

2. 10 pts. Determine the region in the xy -plane for which the initial-value problem

$$(y - x)y' = y + x, \quad y(x_0) = y_0$$

must have a unique solution.

3. 10 pts. In the theory of learning, the rate at which a subject is memorized is assumed to be proportional to the amount that is left to be memorized. Suppose M denotes the total amount of a subject to be memorized and $A(t)$ is the amount memorized at time $t > 0$. Determine a differential equation for $A(t)$.

4. 10 pts. Solve the differential equation

$$\sin 3x + 2y \cos^3 3x \frac{dy}{dx} = 0.$$

5. Consider the initial-value problem

$$\frac{dy}{dx} = \frac{3x - 1}{4y}, \quad y(-2) = -1.$$

- (a) 10 pts. Find the explicit solution to the initial-value problem.
(b) 5 pts. Find the solution's interval of validity by analytical means.

6. 10 pts. Solve the linear equation:

$$\frac{dy}{dx} + 2xy = y + 4x - 2.$$

7. 10 pts. Solve the initial value problem:

$$L \frac{di}{dt} + Ri = E, \quad i(0) = i_0,$$

where L , R , E , and i_0 are constants.

8. 10 pts. Solve the exact equation with given initial condition:

$$e^x + y + (2 + x + ye^y)y' = 0, \quad y(0) = 1.$$

9. 10 pts. Find an integrating factor of the form

$$\mu(x) = \exp\left(\int \frac{M_y - N_x}{N} dx\right) \quad \text{or} \quad \mu(y) = \exp\left(\int \frac{N_x - M_y}{M} dy\right)$$

so as to make

$$y(x + y + 1) + (x + 2y)\frac{dy}{dx} = 0$$

an exact equation, and then solve it.

10. 10 pts. Solve the homogeneous differential equation:

$$x + ye^{y/x} - xe^{y/x}y' = 0.$$

11. 10 pts. each Initially 100 mg of a radioactive isotope was present. After 6 hours the mass of the isotope had decreased by 3.4%.

- (a) If the rate of decay of the isotope is proportional to the mass present at time t , find the mass remaining after 24 hours.
- (b) When will 98% of the isotope be gone?

12. 15 pts. A dead body was found in a house where the temperature was a constant 68° F. At the time of discovery the core temperature of the body was determined to be 83° F. One hour later a second measurement showed the core temperature to be 77° F. Assume the core temperature at the time of death was 98.6° F. How much time elapsed between the time of death and the time the body was found?

13. 15 pts. A large tank is partially filled with 400 liters of water in which 4 kilograms of sugar is dissolved. Water containing 0.04 kg of sugar per liter is pumped into the tank at a rate of 18 L/min. The well-mixed solution is meanwhile pumped out at a slower rate of 15 L/min. Find the number of kilograms of sugar in the tank at time t .

A couple trigonometric identities: $\sin(2\theta) = 2 \sin \theta \cos \theta, \quad \cos(2\theta) = 2 \cos^2 \theta - 1.$