MATH 250
SUMMER 2016 NAME:
EXAM 3

1. $\boxed{\text{10 pts.}}$ Rewrite the expression

$$\sum_{n=2}^{\infty} n(n-1)c_n x^{n-2} - 5\sum_{n=0}^{\infty} c_n x^{n+2}$$

using a single power series whose general term involves x^n.

2. $\boxed{\text{15 pts.}}$ Using the power series method with center at 0, find the general solution to $y' = xy$ the power series method.

3. $\boxed{\text{20 pts.}}$ Use the power series method with center at 0 to solve the initial-value problem

$$y'' - 2xy' + 8y = 0, \quad y(0) = 3, \quad y'(0) = 0.$$

4. $\boxed{\text{10 pts.}}$ Use the definition of the Laplace transform to find $\mathcal{L}[f]$ for

$$f(t) = \begin{cases} 1, & t \in [0, 8) \\ t, & t \in [8, \infty) \end{cases}$$

5. $\boxed{\text{5 pts. each}}$ Use the table provided to find the Laplace transform.

   (a) $f(t) = -2t^5$

   (b) $g(t) = 4\cos^2 4t$

   (c) $h(t) = e^t \sinh t$

6. $\boxed{\text{10 pts.}}$ Find the inverse Laplace transform:

$$\mathcal{L}^{-1}\left[\frac{1}{s^2 + s - 20}\right](t).$$

7. $\boxed{\text{20 pts.}}$ Use the method of Laplace transforms to solve

$$y'' - 4y' + 4y = t^2, \quad y(0) = 1, \quad y'(0) = 0.$$

8. $\boxed{\text{20 pts.}}$ Use the method of Laplace transforms to solve

$$y'' + 4y = f(t), \quad y(0) = 0, \quad y'(0) = -1$$

where

$$f(t) = \begin{cases} 2, & 0 \le t < 1 \\ 0, & t \ge 1. \end{cases}$$

9. $\boxed{\text{15 pts.}}$ Use the method of Laplace transforms to solve

$$y' + y = \delta(t-3), \quad y(0) = 2.$$

$f(t)$	$\mathcal{L}[f](s)$	$\mathrm{Dom}(\mathcal{L}[f])$
$t\sin bt$	$\dfrac{2bs}{(s^2+b^2)^2}$	$s>0$
$t\cos bt$	$\dfrac{s^2-b^2}{(s^2+b^2)^2}$	$s>0$
$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$	$s>a$
$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$	$s>a$
$e^{at}t^n,\ n=0,1,\ldots$	$\dfrac{n!}{(s-a)^{n+1}}$	$s>a$
$u(t-a),\ a\geq 0$	$\dfrac{e^{-as}}{s}$	$s>0$
$\delta(t-a),\ a\geq 0$	e^{-as}	$s>0$
$(f*g)(t)$	$\mathcal{L}[f(t)](s)\mathcal{L}[g(t)](s)$	$s>0$

$\mathcal{L}[f'](s)=s\mathcal{L}[f](s)-f(0)$

$\mathcal{L}[f''](s)=s^2\mathcal{L}[f](s)-sf(0)-f'(0)$

$\mathcal{L}[t^nf(t)](s)=(-1)^nF^{(n)}(s)$

$\mathcal{L}[f(t-a)u(t-a)](s)=e^{-as}\mathcal{L}[f(t)](s)$

$\mathcal{L}[g(t)u(t-a)](s)=e^{-as}\mathcal{L}[g(t+a)](s)$

$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

$\cos^2 x=\frac{1}{2}(1+\cos 2x)$

$\sin x\cos y=\frac{1}{2}[\sin(x+y)+\sin(x-y)]$

$\cos x\cos y=\frac{1}{2}[\cos(x+y)+\cos(x-y)]$

$\sin x\sin y=\frac{1}{2}[\cos(x-y)-\cos(x+y)]$